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THE DESIGN OF PRESSURE VESSELS FOR VERY HIGH PRESSURE OPERATION

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LIST OF SYMBOLS

σ	Stress
σ_θ	Tangential or hoop stress
σ_r	Radial stress
σ_z	Axial or longitudinal stress
ϵ	Strain
ϵ_θ	Tangential strain
ϵ_r	Radial strain
ϵ_z	Axial strain
r_1	Radius of bore
r_2	Radius of outer diameter
r	Some intermediate radius
K	Diameter ratio, r_2/r_1
K_t	Total diameter ratio
ρ	Radius of elastic-plastic boundary
n	Plastic diameter ratio, ρ/r_1
σ_y	Yield stress in tension
σ_u	Ultimate tensile strength
τ_y	Yield stress in shear
E	Young's elastic modulus
ν	Poisson's ratio
u	Radial displacement
P_i	Internal pressure
P_o	External pressure

P_{12}	Interface pressure between elements 1 and 2
P_y	Pressure at initial yield or re-yield
P_u	Ultimate or rupture pressure
P_c	Complete overstrain or collapse pressure
P_A	100 percent autofrettage pressure ($\sigma_{\theta R}$ @r_1 = - σ_y)
P^*	Specific value of pressure
t	Wall thickness
δ	Radial interference
α	Semi-angle of sectors of segmented cylinder
γ	Strain hardening exponent

SUBSCRIPTS

e	Elastic region
p	Plastic region
R	Residual

THE DESIGN OF PRESSURE VESSELS FOR VERY HIGH PRESSURE OPERATION

ABSTRACT

This report is a review of the theory and practice of pressure vessel design for vessels operating in the range of internal pressures from 1 to 55 kilobars (approximately 15,000 to 800,000 psi) and utilizing fluid pressure media. The fundamentals of thick walled cylinder theory are reviewed, including elastic and elastic-plastic theory, multi-layer cylinders and autofrettage. The various methods of using segmented cylinders in pressure vessel design are reviewed in detail.

The factors to be considered in the selection of suitable materials for pressure vessel fab are discussed. These factors include strength, toughness and environmental factors. A brief review of the materials currently available is also included.

The report also includes a discussion of pressure seals and closures suitable for use in this pressure range and of methods of supporting the end closures of the vessel.

Cross-Reference Data

Pressure vessels

Design

High pressure

Thick walled Cylinders

Autofrettage

Pressure seals

I. INTRODUCTION

The study of material behavior under pressure is of interest to investigators in a wide variety of disciplines. However, regardless of the specific area of interest, the first requirements of any investigator in this field are a suitable vessel to contain the required pressure and the specific experiment, and a means of generating the pressure.

Static pressures in excess of 150 kb have been produced for the purpose of studying their effect on materials and various physical and chemical phenomena. However, when the pressure exceeds approximately 30 to 40 kb, the stress levels involved and the problems of sealing and solidification of the pressure transmitting media necessitate the use of solid transmitting media systems such as the belt, tetrahedron and various special piston and cylinder or anvil devices. Since the subject of mechanical properties at high pressure normally implies hydrostatic pressure, this discussion of pressure vessels and equipment will be limited to those devices using gaseous or fluid pressure media.

The specific type of high pressure system required by the investigator depends, of course, on the experiment being conducted. It would not be possible nor worthwhile to consider the details of all of the types of systems used or proposed for use. However, all types of

systems consist basically of one or more pressure cylinders along with associated closures, seals, pistons and mechanical or hydraulic force generating systems. Therefore, they are all based on similar principles of design. The main purpose of this report then will be to present the philosophy of pressure cylinder design with brief mention of such subjects as pressure generation, closures and pistons, and materials selection. The aim is to present the information in a form usable to all investigators regardless of their specific pressure system requirements.

II. THEORY OF PRESSURE CYLINDER DESIGN

The design of pressure vessels for operation at very high pressures is a complex problem involving many considerations including definition of the operating and permissible stress levels, criteria of failure, material behavior, etc. For the purpose of developing the design philosophy and the relative operational limitations of various approaches, the elastic strength or yielding pressure of the vessel will be used as the criterion of failure. It should be noted, however, that some designs can be used at pressures in excess of that at which yielding of one or more components is predicted. Generally however, the use of vessels beyond the yielding pressure will depend upon the amount of plastic strain permissible and the ductility of the materials involved.

As a means of presenting the theory of vessel design, the simplest case of a residual stress-free monobloc cylinder will be considered first. Next will be considered how the elastic operating range can be extended by the use of residual stresses which counteract the operating stresses. Finally, means for extending the elastic range by various techniques including segmenting and/or external support will be examined.

A. Monobloc Cylinders Without Residual Stresses

The simplest form of a pressure cylinder is the so-called thin-walled cylinder. Although, rigorously speaking, the only thin-walled cylinder is a cylindrical membrane, a cylinder whose wall thickness is small with respect to its radius may usually be considered as a thin-walled cylinder for design purposes. Under se conditions the tangential stress is assumed to be constant through the wall thickness and the radial stress is considered negligible. The tangential or hoop stress is given by:

$$\sigma_\theta = \frac{P_i r_1}{t} \quad \ldots\ldots\ldots\ldots (1)$$

If the cylinder has closed ends, the longitudinal stress is given by:

$$\sigma_Z = \frac{P_i r_1}{2t} \quad \ldots\ldots\ldots\ldots (2)$$

For cylinders of any significant wall thickness, the above equations are approximate and should be used with caution. In case of any question as to applicability of these equations, the thick-walled cylinder equations, to be presented later, should be utilized. Since the maximum operating pressure for a thin-walled cylinder is less than 0.5 kb which is well below the range of interest of this report, all further discussion of pressure cylinders will be limited to thick-walled cylinders.

For operating pressures in the range of interest of this report, i.e., greater than 1 kb, the pressure vessel utilized will virtually always be some form of a thick-walled cylinder. The elastic stresses

in such a cylinder containing no residual stresses are given by the well-known Lamé equations. These equations are obtained by solution of the differential equation obtained by summing forces in the radial direction on a differential element of the cylinder. This derivation is given by most standard texts on advanced strength of materials[1] and yields the following equations for the stresses resulting from internal pressure:

$$\sigma_\theta = \frac{P_i}{K^2 - 1}\left(1 + \frac{r_2^2}{r^2}\right) \qquad \ldots\ldots (3)$$

$$\sigma_r = \frac{P_i}{K^2 - 1}\left(1 - \frac{r_2^2}{r^2}\right) \qquad \ldots\ldots (4)$$

These equations apply regardless of the condition of longitudinal stress or end condition since they are derived directly from equilibrium of forces in the radial direction and the longitudinal stress has no component in the radial direction.

The longitudinal stress for the closed end condition (i.e., the pressure force on the end closures is supported by the cylinder wall) is given by:

$$\sigma_Z = \frac{P_i}{K^2 - 1} \qquad \ldots\ldots (5)$$

The open end condition is that in which the pressure force on the end closures is supported by some external structure such as a press and in which the ends of the cylinder are free to move in the longitudinal direction. For an elastic cylinder, this results in the plane stress condition where:

$$\sigma_Z = 0 \qquad \ldots\ldots (6)$$

The pressure at which initial yielding will occur at the bore surface of a monobloc cylinder is obtained by the incorporation of the above equations into the appropriate yield criterion. The most generally used yield criteria are the maximum shear stress or Tresca and the strain energy of distortion or Maxwell-Mises.

The Tresca criterion, as related to a thick-walled cylinder, is defined by the following equation:

$$\frac{\sigma_\theta - \sigma_r}{2} = \tau_y \qquad \ldots . (7)$$

However, shear yield strength data for materials is not usually available and it is commonly assumed that $2\tau_y = \sigma_y$. In terms of the tensile yield stress then, the Tresca criteria become:

$$\sigma_\theta - \sigma_r = \sigma_y \qquad \ldots . (8)$$

Since for both the open and closed end condition the longitudinal stress is the intermediate principal stress, the yield pressure is obtained by substituting eqs. (3) and (4) with $r = r_1$ into eq. (8) which gives:

$$\frac{P_Y}{\sigma_y} = \frac{K^2 - 1}{2k^2} \qquad \ldots . (9)$$

The Maxwell-Mises yield criterion is defined by the following equation:

$$(\sigma_\theta - \sigma_r)^2 + (\sigma_r - \sigma_Z)^2 + (\sigma_Z - \sigma_\theta)^2 = 2\,\sigma_y^{\,2} \qquad \ldots (10)$$

Substituting into eq. (10), eqs. (3) and (4) with $r = r_1$, and either eq. (5) or (6) gives the yield pressure for the closed and open end condition respectively:

$$\frac{P_y}{\sigma_y} = \frac{K^2 - 1}{\sqrt{3}K^2} \qquad \ldots\ldots (11)$$

and

$$\frac{P_y}{\sigma_y} = \frac{K^2 - 1}{\sqrt{3K^4 + 1}} \qquad \ldots\ldots (12)$$

These equations are plotted in Fig. 1. Experimental results have shown that the Maxwell-Mises criterion as given by eqs. (11) and (12) most accurately predict the actual yielding behavior of high strength alloy steel cylinders.[2]

Crossland et al[3] have shown that good agreement with experimental results can be obtained for the closed end case by using eq. (7) and a value of τ_y determined from a torsion test. This gives:

$$\frac{P_y}{\tau_y} = \frac{K^2 - 1}{K^2} \qquad \ldots\ldots (13)$$

Equation (13) is consistent with eq. (11) if the relationship between σ_y and τ_y as predicted by the Maxwell-Mises yield criterion, namely, $\sigma_y = \sqrt{3}\,\tau_y$, is utilized.

It can readily be seen that the strength of a monobloc cylinder is not significantly affected by the end condition, particularly in the large diameter ratios and in the absence of any externally applied longitudinal forces.

For diameter ratios greater than three, the values of bore stress decrease only slightly with increasing diameter ratio. Therefore, in the best case, the maximum yield pressure is only slightly greater than one-half the yield strength, regardless of diameter ratio. This can be seen

in the lower curve of Fig. 3. The effects of exceeding the yield pressure, and the fracture or rupture characteristics of monobloc cylinders will be subsequently discussed.

The operating pressure limitations of a monobloc cylinder can be overcome partly by some technique which will produce a compressive tangential stress in the material at the bore. This will tend to counteract the high tensile stress due to internal pressure and thus increase the pressure required to produce yielding.

The two general approaches that can be utilized to produce this compressive stress are residual stresses and/or variable external restraint, which will now be discussed.

B. Residual Stresses

Many techniques have been utilized to produce favorable residual stresses but only three have been used with any amount of success. They are multi-layer construction, autofrettage and wrapping.

The latter technique consists of wrapping a thick-walled cylinder with a high strength filament such as wire, tape, or fiberglass under tension to produce the desired compressive residual stress in the cylinder. This technique has the advantage of using very high strength materials that are often not available in other than wire or filament form. It has not been extensively used in recent years, due primarily to economic factors and problems in accurate control of the residual stress magnitude. However, recent studies based on new developments in fiberglass filament winding indicate that fiberglass filament winding of autofrettaged cylinders may offer significant advantages when vessel weight is a factor. However, this technique has not been developed to a point of practical usefulness, and since weight is not usually a serious problem, it will not be further discussed.

The remainder of this discussion on techniques utilizing residual stresses will, therefore, be restricted to multi-layer and autofrettage.

1. Multi-layer Cylinders

Multi-layer cylinders are assembled so as to have an interference fit between the respective layers. This will result in compressive residual stresses in the inner element and tensile residual stresses in the outer element. The interference fit between layers may be accomplished by heating and "shrinking on" of outer elements having a bore diameter slightly smaller than that of the respective inner element or by having matched tapers on the inside and outside surface of the outer and inner elements respectively and forcing the elements into each other by means of a press.

The resultant interface pressures, residual stresses, operating stresses and the yielding pressure and location of the onset of yielding will be a function of several variables. The most pertinent of these variables are the number of elements and their relative yield strengths and elastic moduli, and the diameter ratios of the elements. As an approach to presenting the design theory for multi-layer vessels, the simplest case of a two element construction having equal strengths and elastic moduli in the inner (liner) and outer (jacket) elements will be considered first. Following this, the two-element cylinder, wherein the yield strengths and elastic moduli differ between jacket and liner, will be discussed. Finally, the question of a cylinder consisting of more than two layers will be considered.

a. Two-element cylinders - same material. The simplest and most generally used form of the multi-layer vessel involves two elements consisting of a jacket and liner. For the case where the yield strength and elastic moduli of the liner and jacket are the same, the resulting interface pressure (P_{12}) between the two elements, as given by Timoshenko[1], is:

$$P_{12} = \frac{E\,\delta}{r_2} \; \frac{(r_2^2 - r_1^2)\,(r_3^2 - r_2^2)}{2r_2^2\,(r_3^2 - r_1^2)} \qquad \ldots . (14)$$

where subscript $_1$ refers to the bore of the inner cylinder, subscript $_2$ refers to the interface and subscript $_3$ refers to the outside of the outer cylinder. δ is the initial radial interference prior to assembly.

The resulting residual stresses can be determined from the above interface pressure and the Lamé equations. The residual stresses in the inner cylinder are given by:

$$\sigma_{\theta 1} = \frac{-P_{12}\,K_1^2}{(K_1^2 - 1)} \left(1 + \frac{r_1^2}{r^2}\right) \qquad \ldots . (15)$$

$$\sigma_{r1} = \frac{-P_{12}\,K_1^2}{(K_1^2 - 1)} \left(1 - \frac{r_1^2}{r^2}\right) \qquad \ldots . (16)$$

where K_1 is the diameter ratio of the inner cylinder. The stresses in the outer cylinder are given by eqs. (3) and (4) letting $K = K_2$, $P_i = P_{12}$ and using r_1 and r_2 for the outer cylinder.

To determine the elastic stress state in the composite cylinder when subjected to an internal pressure, the stresses produced by the pressure are added to the residual stresses as given in eqs. (15)

and (16). The pressure stresses are computed from eqs. (3) and (4) where K equals the total diameter ratio for the composite cylinder (r_3/r_1).

The yield pressure for a composite cylinder is obtained by substitution of the sum of the pressure and residual stresses into the appropriate yield criterio as were given in eqs. (8) or (10). Based on the work of Manning[4], the optimum design of a two element vessel is where the diameter ratio of the liner and jacket are equal and the interface pressure is such as to result in simultaneous yielding at the bore of the liner and jacket at the maximum elastic operating pressure. This gives for the condition of yielding at the bore of the outer layer:

$$\sigma_y = P_{12} \frac{2K}{K - 1} + P_i \frac{2K}{K^2 - 1} \qquad \ldots . (17)$$

Solving this equation for the interface pressure yields:

$$P_{12} = \sigma_y \frac{K - 1}{2K} - P_i \frac{K - 1}{K^2 - 1} \qquad \ldots . (18)$$

The condition of yielding at the bore of the inner element is given by:

$$\sigma_y = P_i \frac{2K^2}{K^2 - 1} - P_{12} \frac{2K}{K - 1} \qquad \ldots . (19)$$

Substituting P_{12} from eq. (18) into eq. (19) yields:

$$P_i = \sigma_y \frac{K - 1}{K} \qquad \ldots \ldots (20)$$

In terms of the Maxwell-Mises yield criterion eq. (10), the pressure required to produce yielding has been developed by the authors[5] and is given by:

$$\frac{P_y}{\sigma_y} = \frac{2QR + \sqrt{Q^2 + .75 - 3R^2}}{Q^2 + .75} \quad \ldots\ldots (21)$$

where: $Q = \frac{1}{2} + \frac{K + 1}{K - 1}$

$$R = \frac{K}{\sqrt{3K^2 + 1}}$$

and K is the total diameter ratio. This equation is shown plotted in Fig. 3 where the elastic strength of a two-element cylinder may be compared with that of a monobloc cylinder having no residual stresses.

This analysis has assumed that there are no longitudinal stresses, i.e., $\sigma_z = 0$. Residual longitudinal stresses may result from the shrinking operation and, in some cases, may be of significant magnitude. These stresses are a function of the length of the cylinders, the coefficient of friction at the interface, and the way in which the assembly operation is performed. Longitudinal pressure stresses will exist in a closed end cylinder and their distribution will depend on the method in which the end closures are attached to the vessel. In view of the many variables and unknown factors involved with a determination of these longitudinal stresses, no detailed analysis of their effect will be presented.

In general, the effect of longitudinal stress on the yielding pressure of a thick-walled cylinder is very small, as was shown earlier.

Neglecting the longitudinal stresses in the above analysis will probably not result in any significant error in yielding pressure. However, in some cases, particularly long vessels composed of relatively thin walled component cylinders, it may be necessary to consider longitudinal stresses in a critical design.

b. Two-element cylinders - different materials. The elastic solutions for the interface pressure (P_{12}) and initial radial interference (δ), as given by eq. (14), assumed identical elastic constants for the inner and outer cylinders. In addition, the yielding behavior, as treated in eqs. (20) and (21), assumed identical yield strengths for the two cylinders. The case in which the elastic constants and yield strengths are not identical will now be considered.

Pugh[6] has considered the effects of different yield strengths and elastic constants. Using the Tresca yield condition he determined the following equation for yielding of an optimum, two-element cylinder:

$$\frac{P_y}{\sigma_{y2}} = \frac{1}{2}\left(\frac{\sigma_{y1}}{\sigma_{y2}} + 1\right) - \frac{1}{K}\sqrt{\frac{\sigma_{y1}}{\sigma_{y2}}} \qquad \ldots\ldots (22)$$

The condition of an optimum cylinder design is:

$$K_1 = \left(\frac{\sigma_{y1}}{\sigma_{y2}}\right)^{\frac{1}{4}} \sqrt{K} \qquad \ldots\ldots\ldots\ldots (23)$$

and for the initial radial interference (δ)

$$\frac{\delta}{r_2} = \frac{1}{2}\left[\sigma_{y2} - \frac{1}{K}\sqrt{\sigma_{y1}\,\sigma_{y2}}\right]\left[\frac{1-\nu_1^2}{E_1}\left(\frac{K_1^2+1}{K_1^2-1}\right) + \frac{1-\nu_2^2}{E_2}\left(\frac{K_2^2+1}{K_2^2-1}\right) - \frac{\nu_1(1+\nu_1)}{E_1} + \frac{\nu_2(1+\nu_2)}{E_2}\right]$$

$$- \frac{1-\nu_1^2}{E_1}\left[\frac{\sigma_{y1} + \sigma_{y2} - \frac{2}{K}\sqrt{\sigma_{y1}\,\sigma_{y2}}}{K_1^2 - 1}\right] \qquad \ldots\ldots (24)$$

It can be seen from eq. (24) that slight variations in elastic constants E and ν can have a significant effect on the design parameters

for a compound cylinder. Variations in yield strength will also have considerable effect as would be expected.

Crossland and Burns[7] experimentally examined the pertinent design parameters, such as shrinkage stresses and yielding characteristics of nominally optimum designed, two-element cylinders. They observed good agreement with the theoretical solutions discussed thus far. However, they did observe the existence of a significant longitudinal shrinkage stress. They also presented an analysis of the behavior of the cylinder subsequent to yielding and compared the experimental pressure-expansion curves with this analysis well into the plastic region.

c. Multi-layer cylinders having more than two elements.

The above analysis of a compound cylinder has considered only a two-element cylinder. The analysis of a multi-element cylinder is basically the same. The residual stresses produced by each additional element are calculated as above and added to those produced by prior elements. To determine the final yield pressure, the total stress state at the bore of each element is calculated and checked for yielding. The analysis of a compound cylinder having more than two elements has been investigated by Becker et al[8-10] and by Pugh[6], using the Tresca yield criteria. The solutions of these investigators will now be summarized.

The pressure difference across an n'th element at yield at the bore of the element, based on eq. (9), is:

$$P_n - P_{n+1} = \frac{\sigma_{yn}}{2}\left(1 - \frac{1}{K_n^2}\right) \quad \ldots\ldots (25)$$

If there are "m" elements and no external pressure on the outer element, i.e., $P_{m+1} = 0$, the internal pressure to produce simultaneous yielding in all cylinders is:

$$P_y = \sum_{n=1}^{m} \frac{\sigma_{yn}}{2}\left(1 - \frac{1}{K_n^2}\right) \quad \ldots\ldots (26)$$

The maximum value of P_y is found by differentiating eq. (26) with respect to K_n and equating to zero, using the relationship $K_1.K_2.K_3. \ldots\ldots K_n = K_t$.

$$\frac{\sigma_{y1}}{K_1^2} = \frac{\sigma_{y2}}{K_2^2} \ldots = \frac{\sigma_{yn}}{K_n^2} = \frac{1}{\beta^2} \quad \ldots\ldots (27)$$

where

$$\beta^m = \frac{K_t}{(\sigma_{y1}\ \sigma_{y2}\ \cdot\cdot\cdot\ \sigma_{ym})^{\frac{1}{2}}} \qquad \ldots . (28)$$

therefore, the maximum yield pressure is:

$$P_{y\ max} = \frac{1}{2} \sum_{n=1}^{m} \sigma_{yn} - \frac{m}{2}\ \frac{\sigma_{y1}}{K_1^{\ 2}} \qquad \ldots (29)$$

If all elements have the same yield strength, this becomes:

$$\frac{P_{y\ max}}{\sigma_y} = \frac{m}{2}\left(1 - \frac{1}{K_t^{\ 2/m}}\right) \qquad \ldots . (30)$$

The upper limit to the above analysis occurs when the compressive residual stress at the bore of the inner cylinder due to the shrinkage stresses produced by adding an outer element exceeds the yield strength of the material. This will occur when there are more than two elements and when the yield pressure calculated from eq. (29) exceeds:

$$P_y = \sigma_{y1}\ \frac{K_t^{\ 2} - 1}{K_t^{\ 2}} \qquad \ldots . (31)$$

for the case of equal elastic constants. The derivation of eq. (31) is the same as that for reverse yielding of an autofrettaged cylinder which will be subsequently discussed.

As an upper limit to the design of multi-layer, compound cylinders, it can be shown that the strength of an optimum design, multi-layer vessel approaches that of an ideal, autofrettaged cylinder of the same material, strength level and total diameter ratio as the number of layers approaches infinity.

2. Autofrettage

The second method for inducing residual stresses in a cylinder is autofrettage. If a monobloc, thick-walled cylinder is subjected to an internal pressure exceeding its yield pressure, plastic deformation will initiate at the bore and, as the pressure is increased, will proceed through the cylinder wall until the plastic-elastic interface reaches the outside surface. At this point, the cylinder material is in a completely plastic state which is defined as complete overstrain, and the associated pressure referred to as the complete overstrain or collapse pressure. The effect of exceeding this pressure and the problem of rupture will be discussed later under "Effect of Material Behavior".

Due to elastic recovery, when the internal pressure is released after either partial or complete overstrain, the material near the outside surface, which has been deformed the least amount, will attempt to return to its original diameter and the material near the bore, which has been deformed the most, will attempt to remain deformed. This results in a residual compressive stress at the bore and a residual tensile stress at the outside surface with a gradual transition through the wall thickness. The process of producing this residual stress distribution by means of plastic deformation of the cylinder by internal pressure is known as autofrettage. The process has been utilized for many years in the manufacture of gun barrels and in recent years has been applied to pressure vessels designed to operate at very high pressures.

A study of the available literature on the theoretical solutions for the overstrain of thick-walled cylinders can result in considerable confusion on the part of the reader. The problem has been studied by a large number of investigators and there is considerable disagreement in the results obtained. The reason for this can be seen by considering assumptions which must be made concerning materials behavior, yield criteria, end condition, stress and strain distribution, etc., in order to obtain an analytical solution of the partially plastic, thick-walled cylinder problem.

The problem is further compounded by the fact that the selection of some combinations of these assumptions results in mathematical problems which are either impossible to solve or must be solved by numerical techniques. As a result of these difficulties, several empirical or semi-empirical solutions based on experimental data have been proposed.

A full analysis and comparison of many solutions based on various combinations of assumptions is not considered to be within the scope of this text. However, some comments on validity and significance of the possible assumptions and a few resultant solutions will be presented.

a. Analysis of assumptions.

Material behavior - The rigid-perfectly plastic material model may be a fairly accurate approximation in problems involving large plastic strains in which the elastic strains are small with respect to the plastic strains. In a partially plastic, thick-walled cylinder the plastic strains are of the same order of magnitude as the elastic strains and, therefore, the elastic strains cannot be neglected. Most high strength steels which are currently utilized for very high pressure vessels exhibit very little strain hardening in the region of small plastic strains and can thus be quite accurately represented by an elastic-perfectly plastic model. The use of a strain hardening model introduces serious difficulties in the mathematical analysis, particularly when combined with the Maxwell-Mises yield criterion. A method of avoiding this difficulty has been developed by Manning[11] and will be subsequently discussed.

Yield condition - Although there have been a number of yield criteria proposed over the years, only two predict the behavior of a moderately ductile steel, as would normally be used in a pressure vessel, with any degree of accuracy. The Maxwell-Mises or octahedral shear stress criterion has been shown to be the most accurate for all combinations of triaxial stress and should be used wherever possible. However, its use results in considerable mathematical difficulties. The Tresca or maximum shear stress criterion is often used for design of structures and machines. It is always conservative with respect to the Maxwell-Mises criterion if the yield loci are assumed to coincide

on the principal stress axis, i.e., if they are based on the same yield stress in tension. Its application to the thick-walled cylinder problem results in a comparatively simple mathematical formulation as will be shown later.

End condition - In practice, pressure vessels are autofrettaged in both open and closed end conditions. The restrained end condition does not actually exist in practice. However, since the assumption associated with the restrained end condition (i.e., $\epsilon_Z = 0$) provides a significant simplification of the mathematics, it is sometimes used to obtain an approximate solution.

Longitudinal stress distribution - The lack of knowledge of the distribution of longitudinal stress across the wall thickness is a significant limitation to the development of a rigorous thick-walled cylinder theory. From the Lamé equations it can be shown that the longitudinal stress is constant in an elastic cylinder. However, when the cylinder becomes partially plastic, the difference in Poisson's ratio between the elastic and plastic zones may result in a variation in longitudinal stress across the wall. In order to determine this stress distribution some plastic stress-strain law, such as the Prandtl-Reuss equations, must be utilized in the plastic zone. In order to avoid the mathematical difficulties and uncertainties of this type of solution, many investigators have either assumed a constant longitudinal stress throughout the wall, or a constant stress in the elastic region and some assumed distribution, such as linear or logarithmic, in the plastic region. For any assumed distribution, the condition that the integral

of the longitudinal stress over the wall thickness equals the total longitudinal force on the cylinder as determined by the end condition must be satisfied.

Longitudinal strain distribution - It is generally assumed that the longitudinal strain is constant across the wall thickness. The result of assuming a significant variation in longitudinal strain across the wall thickness, in a long cylinder, would be a substantial variation in the change of overall length. This would produce distortions of the end surfaces of the cylinder which have not been observed in actual practice. The actual magnitude of longitudinal strain is, of course, determined by the end condition.

Compressibility - The assumption of incompressibility of the material in both the plastic and elastic regions has been used by several early investigators to obtain closed form solutions of the problem. However, since there is considerable volume change associated with elastic strains and since the elastic strains are a significant part of the total strain in a partially plastic cylinder, the errors resulting from this assumption may be significant. Actually, however, some solutions based on this assumption have been shown to be in fair agreement with more rigorous solutions and experimental results.

The assumption of a compressible material in the elastic region and incompressibility in the plastic region would appear to give a better approximation. However, this results in a severe discontinuity at the elastic-plastic interface. Since continuity at this interface

is usually a required boundary condition for the solution of the problem, this assumption is not generally usable.

The material in the plastic region of the cylinder, prior to entering the plastic state, must be subjected to elastic strains up to the yield point of the material. After this material becomes plastic, it still contains the volume changes associated with the elastic stresses. The most accurate solution is thus obtained by assuming elastic compressibility throughout the cylinder.

b. Available solutions. The basic concept of autofrettage was proposed by St. Venant[12]. The extensive pioneering experimental studies of the overstrain of thick-walled cylinders by such investigators as Cook and Robertson[13], Langenberg[14] and Macrae[15] led to the wide utilization of the autofrettage process for gun tube manufacture, particularly during the World War II era.

Following the war a large number of papers appeared which presented considerable experimental data and a number of analytical approaches to the elastic-plastic thick-walled cylinder problem. Some of these analytical solutions will be discussed later in this section. A complete survey of these papers will not be attempted here. Reviews are included in a paper by Manning[16] and one by Crossland and Bones[17].

Manning[11] developed a solution for the overstrain of thick-walled cylinders by using the assumptions of plane strain and constancy of volume but considering strain hardening. This was done by noting that the stress condition in a closed end cylinder is equivalent to pure shear with a superposed hydrostatic tensile stress. This permits the conclusion that the relationship between the maximum shear stress and the maximum shear strain is the same in the cylinder as it is in a torsion test. The shear stress-shear strain results obtained from a torsion test can be directly used to predict the pressure-strain response of a cylinder of the same material up to the ultimate pressure. He also presented experimental results for a variety of materials which were in good agreement with his analytical results.

Crossland and Bones[17] and Crossland, Jorgensen and Bones[3] showed good agreement of extensive experimental data on a variety of materials with analytical results based on Manning's approach. Morrison and Franklin[18] also published experimental results which were in reasonable agreement with their analysis using shear stress-shear strain data from torsion tests and assuming that the shear strain in the cylinder is inversely proportional to the square of the radius.

The above analyses and experimental results pertained to the case of low strength, strain hardening materials. However, most of the materials being used today for ultra-high pressure vessels exhibit relatively small strain hardening tendencies. Thus, strain hardening can be neglected which permits a considerably more analytical approach to the elastic-plastic cylinder problem. A few such approaches will now be considered.

As stated previously, a relatively simple solution to the partially plastic thick-walled cylinder problem may be obtained by assuming the Tresca yield criteria and an elastic-perfectly plastic material. If we assume that the longitudinal stress is the intermediate principal stress, a solution for the radial and tangential stresses can be obtained which is independent of end condition or longitudinal stress and strain distribution. Koiter[19] has shown this assumption to

be valid for diameter ratios less than about 5.0 to 6.0 for all end conditions.

For the partially plastic cylinder, the stresses in the elastic region are obtained from the Lamé equations (eqs. (3) and (4)) combined with the fact that the radial stress at the elastic-plastic boundary ($r = \rho$) equals the yield pressure for the elastic portion of the cylinder. This yields:

$$\sigma_{re} = -\frac{\sigma_y \rho^2}{2r_2^2}\left(\frac{r_2^2}{r^2} - 1\right) \qquad \text{. . . . (32)}$$

$$\sigma_{\theta e} = \frac{\sigma_y \rho^2}{2r_2^2}\left(\frac{r_2^2}{r^2} + 1\right) \qquad \text{. . . . (33)}$$

The stresses in the plastic region can now be determined by combining the yield condition eq. (8) and the equilibrium equation:

$$\sigma_\theta - \sigma_r - r\frac{d\sigma_r}{dr} = 0 \qquad \text{. . . . (34)}$$

The resulting differential equation can be integrated directly and solved by applying the boundary conditions

$$@\ r = r_1, \quad \sigma_r = -P_i$$

$$@\ r = \rho, \quad \sigma_{rp} = \sigma_{re}$$

This yields:

$$\sigma_{rp} = -\sigma_y\left[\ln\frac{\rho}{r} + \frac{r_2^2 - \rho^2}{2r_2^2}\right] \qquad \text{. . . (35)}$$

$$\sigma_{\theta p} = \sigma_y\left[-\ln\frac{\rho}{r} + \frac{r_2^2 + \rho^2}{2r_2^2}\right] \qquad \text{. . . (36)}$$

The internal pressure required to produce plastic flow to a depth ρ is therefore given by:

$$P_i = \sigma_y \left[\ln \frac{\rho}{r_1} + \frac{r_2^2 - \rho^2}{2r_2^2} \right] \qquad \ldots\ldots (37)$$

The pressure to produce complete plastic flow or complete overstrain is given by eq. (37) setting $\rho = r_2$. This yields:

$$P_c = \sigma_y \ln K \qquad \ldots\ldots\ldots\ldots (38)$$

This relationship is shown plotted in Fig. 2.

In order to determine the strains or displacements, some assumption must be made concerning the longitudinal stresses and strains. For the purpose of this approximate analysis, the longitudinal stress will be assumed constant throughout the wall. Experimental results[2] indicate that the error in the tangential strain resulting from this assumption is negligible except in the case of very large diameter ratios and large amounts of overstrain.

Assuming elastic compressibility throughout the cylinder, elastic compressibility in the plastic region is expressed by:

$$\epsilon_\theta + \epsilon_r + \epsilon_z = \frac{1 - 2\nu}{E} (\sigma_\theta + \sigma_r + \sigma_z) \qquad \ldots (39)$$

Expressing ϵ_θ and ϵ_r in terms of the radial displacement u, i.e., $\epsilon_\theta = \frac{u}{r}$ and $\epsilon_r = \frac{du}{dr}$ substituting σ_θ and σ_r from eqs. (35) and (36) yields:

$$\frac{du}{dr} + \frac{u}{r} = \frac{(1 - 2\nu)\sigma_y}{E} \left(2 \ln \frac{r}{\rho} + \frac{\rho^2}{r_2^2}\right) + \frac{(1 - 2\nu)}{E} \sigma_z - \epsilon_z \qquad (40)$$

Since σ_z and ϵ_z are assumed independent of r, this equation may be solved using an integrating factor which yields:

$$\left(\frac{u}{r}\right)_p = \frac{(1-2\nu)\sigma_y}{E}\left(\ln\frac{r}{\rho} - \frac{1}{2}\right) + \frac{(1-2\nu)}{2E}\left(\frac{\sigma_y \rho^2}{r_2^2} + \sigma_z\right) - \frac{\epsilon_z}{2} + \frac{C}{r^2} \qquad (41)$$

where C is a constant of integration.

To solve for this constant we use the condition of continuity of displacement at the elastic-plastic boundary. From eqs. (32) and (33) and Hooke's law, the tangential strain in the elastic region is

$$\left(\frac{u}{r}\right)_e = \epsilon_{\theta e} = \frac{1}{E}\left\{\frac{\sigma_y \rho^2}{2r_2^2 r^2}\left[(1+\nu)\, r_2^2 + (1-\nu)\, r^2\right] - \nu\sigma_z\right\} \qquad (42)$$

By setting $r = \rho$ in eqs. (41) and (42) and equating the two equations, C can be determined.

The solution of the problem then becomes:

$$\left(\frac{u}{r}\right)_p = \frac{\sigma_y}{2E}\left[(1-2\nu)\left(\ln\frac{r^2}{\rho^2} - 1\right) + (1-2\nu)\frac{\rho^2}{r_2^2} + (2-\nu)\frac{\rho^2}{r^2} + \frac{\nu\rho^4}{r^2 r_2^2}\right]$$

$$+ \frac{\sigma_z}{2E}\left[(1-2\nu) - \frac{\rho^2}{r^2}\right] + \frac{\epsilon_z}{2}\left[\frac{\rho^2}{r^2} - 1\right] \qquad \ldots\ldots (43)$$

The longitudinal stress and strain are now determined from the appropriate end condition to obtain a complete solution.

For the open end case $\sigma_z = 0$ and ϵ_z is determined for the elastic region from eqs. (32) and (33) and Hooke's law. This yields:

$$\epsilon_z = -\frac{\sigma_y \nu \rho^2}{E\, r_2^2} \qquad \ldots\ldots\ldots (44)$$

Since the longitudinal strain is assumed constant throughout the wall, eq. (44) also applies in the plastic region. Substituting $\sigma_z = 0$ and ϵ_z from eq. (44) into eq. (43) yields:

$$\left(\frac{u}{r}\right)_p = \frac{\sigma_y}{2E}\left[(1 - 2\nu)(\ln\frac{r^2}{\rho^2} - 1) + (1 - \nu)\frac{\rho^2}{r_2^2} + (2 - \nu)\frac{\rho^2}{r^2}\right] \quad . . (45)$$

This equation gives the displacements in the plastic region of an open end, partially plastic cylinder while subjected to internal pressure as given by eq. (37). A typical result of eq. (45) is shown in the lower curve of Fig. 4.

For the closed end case, from eqs. (5) and (37):

$$\sigma_z = \frac{\sigma_y}{K^2 - 1}(\ln\frac{\rho}{r_1} + \frac{r_2^2 - \rho^2}{2r_2^2}) \quad . . . . . (46)$$

From eqs. (32), (33) and (46), and Hooke's law:

$$\epsilon_z = \frac{\sigma_y}{E}\left[\frac{1}{K^2 - 1}(\ln\frac{\rho}{r_1} + \frac{r_2^2 - \rho^2}{2r_2^2}) - \frac{\nu\rho^2}{r_2^2}\right] \quad . . . (47)$$

Substituting σ_z and ϵ_z from eqs. (46) and (47) into eq. (43) yields:

$$\left(\frac{u}{r}\right)_p = \frac{\sigma_y}{2E}\left[(1 - 2\nu)(\ln\frac{r^2}{\rho^2} - 1) + (1 - \nu)\frac{\rho^2}{r_2^2} + (2 - \nu)\frac{\rho^2}{r^2} - \frac{2\nu}{K^2 - 1}(\ln\frac{\rho}{r_1} + \frac{r_2^2 - \rho^2}{2r_2^2})\right] \quad . . . . (48)$$

This equation gives the displacements in the plastic region of a closed end, partially plastic cylinder while subjected to internal pressure. A typical result of this equation is shown by the curve marked Davidson, et al in Fig. 5.

A comparison of eqs. (45) and (48) reveals that the difference between strain values for the closed end and open end condition is

greatest at the condition of initial yielding or when $\rho = r_1$ and decreases as ρ approaches r_2. This difference also decreases with increasing diameter ratio. For a diameter ratio of 2.0 the maximum difference is about 5 percent.

It is interesting to note that we may obtain eq. (48) by simply subtracting the product of ν/E times the longitudinal stress from the tangential strain for the open end case.

Allen and Sopwith[20] published a similar solution using the Tresca yield condition. They did not assume a constant longitudinal stress but determined the longitudinal stress distribution in the plastic region using the Hencky "total strain theory" which is an approximation of the Prandtl-Reuss theory. They obtained a solution in closed form although their equations are quite complicated and difficult to use for engineering design applications. Their results for the tangential and radial stresses are identical to those of the above simplified solution.

Their results for radial displacements yield values which are identical to those obtained from eq. (45) for the open case. For the closed end case eq. (48) gives values which are within 3 percent of those given by Allen and Sopwith as shown in Fig. 5.

The only significant difference between the two theories, therefore, is in the distribution of longitudinal stress which is generally not of particular interest in the design of pressure vessels.

Hill, Lee and Tupper[21] gives a solution using the Tresca yield condition and the Prandtl-Reuss, plastic stress-strain law. The solution

of this problem cannot be obtained in closed form but can be obtained by numerical integration. It yields essentially the same results for the tangential and radial stresses and displacements as the above solutions.

Of the available solutions utilizing the Maxwell-Mises yield criteria, probably the most rigorous, mathematically and physically for a given end condition, is that of Prager and Hodge[22] for the restrained end condition. This solution utilizes the Prandtl-Reuss stress-strain relations to obtain the longitudinal stress distribution in the plastic region. This solution results in a set of simultaneous, partial differential equations which cannot be solved in closed form. Using finite difference methods, a numerical solution was obtained for stresses and displacements, for the case of K = 2. An example of these results is shown in Fig. 5.

Sutherland and Weigle[23] have obtained a solution for the closed end cylinder using the Maxwell-Mises criteria. By assuming a logarithmic distribution of longitudinal stress in the plastic region, they were able to obtain a closed form solution for the stresses. The values of radial and tangential stress obtained from this solution are in very close agreement with those of Prager and Hodge. A solution for the strains or displacements was not obtained.

For the case of the open end condition the assumption of constant longitudinal stress throughout the wall results in the plane stress condition. For this condition, a solution for the radial and tangential stresses in the plastic region can be obtained by direct integration of the differential equation obtained by combining the

equilibrium equation, eq. (34), and the Maxwell-Mises yield criterion. For the plane stress case the yield condition may be written in the form:

$$\sigma_\theta = \frac{1}{2}\left(\sigma_r + \sqrt{4\,\sigma_y^2 - 3\,\sigma_r^2}\right) \qquad \ldots\ldots (49)$$

The boundary condition used to obtain the constant of integration is the fact that the radial stress at $r = \rho$ is equal to the yield pressure for the elastic portion of the cylinder from eq.(12), substituting r_2/ρ for K. The resulting equation for the radial stress in the plastic region has been determined by Weigle[24] and is given by:

$$\ln\left\{\left[\frac{\gamma}{\left(1+\sqrt{3}\,\sqrt{\gamma-1}\right)^2}\right]\left[\frac{12\,r_2^4\,\rho^4}{r^4\,(\rho^4+3r_2^4)}\right]\right\} =$$

$$2\sqrt{3}\left\{\tan^{-1}\left[\frac{\rho^2+3r_2^2}{\sqrt{3}\,(r_2^2-\rho^2)}\right] - \tan^{-1}\sqrt{\gamma-1}\right\} \quad \ldots (50)$$

where $\gamma = \dfrac{4\,\sigma_y^2}{3\,\sigma_{rp}^2}$

Based on the above solution Sutherland[25] obtained a solution for the strains in the plastic region using the Prandtl-Reuss stress-strain relationship. The resulting solution is in the form of an integral equation which must be solved by numerical integration.

The complete overstrain pressure may be determined from eq. (50) by substituting, $\rho = r_2$, $r = r_1$ and $\sigma_r = -P_c$. The resulting expression, as shown in Fig. 2, is:

$$2 \ln \frac{K^2 \sqrt{3\gamma_c}}{1 + \sqrt{3\gamma_c - 3}} = \sqrt{3} - 2\sqrt{3} \tan^{-1} \sqrt{\gamma_c - 1} \quad . . . (51)$$

where $\gamma_c = \dfrac{4 \sigma_y^2}{3 P_c^2}$

It has been shown by the authors[2] that this expression can be approximated within 2 percent by an empirical equation derived from experimental data for open end cylinders:

$$P_c = 1.08 \sigma_y \ln K \quad . . . . (52)$$

This relationship is also shown in Fig. 2.

Based on this equation, the authors have developed an approximate theory based on the Maxwell-Mises yield criterion. This development is based on the assumption that eq. (52) will give the pressure differential which would produce complete plastic flow in a cylinder subjected to both internal and external pressure. We now consider a portion of the cylinder between some radius, r, in the plastic region and the elastic-plastic interface ρ. This portion is equivalent to a separate cylinder of inside radius, r, and outside radius, ρ, subjected to an internal pressure, $-\sigma_{rp}$ (@ r = r) and an external pressure, $-\sigma_r$ (@ $r = \rho$). From the above assumption and eq. (52), we may then write, for this portion of the cylinder:

$$\sigma_r (@\ r = \rho) - \sigma_{rp} = 1.08 \sigma_y \ln \frac{\rho}{r} \quad . . . . . (53)$$

Since σ_r (@ $r = \rho$) is equal to $-P_y$ for the elastic portion of the cylinder as given by eq. (12) with $K = r_2/\rho$ we obtain an approximate equation for σ_r in the plastic region.

$$\sigma_{rp} = -\sigma_y \left[1.08 \ln \frac{\rho}{r} + \frac{r_2^2 - \rho^2}{\sqrt{3r_2^4 + \rho^4}} \right] \quad \ldots . (54)$$

This equation is in good agreement with eq. (50) yielding results which are within 2 percent for all conditions checked. From eq. (54) and eq. (34) the tangential stress is given by:

$$\sigma_{\theta p} = \sigma_y \left[-1.08 \ln \frac{\rho}{r} + 1.08 - \frac{r_2^2 - \rho^2}{\sqrt{3r_2^4 + \rho^4}} \right] \quad \ldots (55)$$

Substituting eqs. (54) and (55) and $\sigma_z = 0$ into eq. (39) and proceeding as in the derivation of eq. (45) we obtain the following expression for the displacements in the plastic region of an open end partially plastic cylinder at pressure:

$$\left(\frac{u}{r}\right)_p = \frac{\sigma_y}{E} \left[1.08 (1 - 2\nu) \ln \frac{r}{\rho} + \frac{\rho^2 (1 - \nu) - r_2^2 (1 - 2\nu) + \frac{\rho^2 r_2^2}{r^2} (2 - \nu)}{\sqrt{3r_2^4 + \rho^4}} \right] \quad (56)$$

As example of the results of eq. (56) for $\left(\frac{u}{r}\right)_p = \epsilon_{\theta p}$ is shown as the middle curve in Fig. 4.

It is the opinion of the authors that eqs. (54), (55) and (56) offer a significant increase in accuracy over eqs. (35), (36) and (45) as demonstrated experimentally in Ref. 2. They have been used with very good results for several years by the authors in design calculations related to production autofrettage of a variety of gun tubes and pressure vessels.

Although eqs. (54) - (56) were derived on the basis of an open end cylinder, as shown in Figs. 1,2,4,5 they can be used with

reasonable accuracy for overstrain in closed end cylinders since the effect of the longitudinal stress on the values of tangential and radial stress and strain is small.

To the authors' knowledge there is no published rigorous solution to the partially plastic cylinder problem for either the open end or closed end case which utilizes the Maxwell-Mises yield condition and the Prandtl-Reuss plastic stress-strain law. The few available solutions using both these theories make other simplifying assumptions which could introduce errors of the same magnitude as the error resulting from using the Tresca yield condition.

c. <u>Residual stresses in autofrettaged cylinders</u>. The ability to extend the elastic pressure range of a cylinder by autofrettage depends upon the residual stresses introduced by the overstrain. The problem of determining the upper limit of pressure at which an autofrettaged vessel will operate elastically requires knowledge of the residual stress distribution after the release of the autofrettage pressure. Thus, as was the case for the multi-layer vessel, the elastic stresses produced by the operating pressure are then added to these residual stresses. The pressure which will produce further yielding can then be calculated from these total stresses and the appropriate yield criterion.

As a first approximation in calculating the residual stresses, the cylinder is assumed to recover elastically. The residual stresses may then be determined by simply subtracting the elastic stresses produced by the autofrettage pressure, as given by the Lamé equations, eqs. (3), (4) and either (5) or (6), from the stresses at pressure, as determined from the appropriate elastic-plastic solution.

d. <u>Maximum elastic operating pressure of autofrettaged cylinders</u>. The assumption of purely elastic recovery upon the release of the overstrain pressure requires that the residual stresses not exceed the yield strength of the material in compression. For the case of a completely overstrained open end cylinder with a diameter ratio exceeding approximately 2.2 depending on the theory used, it will be found that the compressive residual tangential stress at the bore, calculated as above, will exceed the yield strength of the material. This will result in compressive plastic flow of the material near the bore on the release of the autofrettage pressure. This phenomenon is known as reverse yielding and results in a residual stress distribution which obviously cannot be calculated by assuming purely elastic recovery. There has been very little analytical work published on the problem of significant amounts of reverse yielding and no theoretical discussion of the resulting residual stress distribution will be included here for reasons which will be discussed later.

From the preceding analysis it can be shown that an autofrettaged cylinder of a diameter ratio less than 2.2 can theoretically be designed to operate elastically at pressures up to the complete overstrain pressure. For greater diameter ratios, due to reverse yielding, the limiting pressure for totally elastic operation is less than the complete overstrain pressure. An approximate value for this limiting pressure may be obtained by the following simple analysis.

Reviewing the basic principle of autofrettage, we observe that, while a cylinder is subjected to a pressure causing partial overstrain,

a condition of yielding exists throughout the plastic region. Since the residual stresses after release of this pressure, if there is no reverse yielding, are obtained by subtracting the elastic stresses associated with this pressure, the stress state on re-application of this same pressure must be identical to that existing on the first application. This means that, if the pressure is now increased, yielding will occur throughout the plastic region simultaneously. Therefore, in order to determine the condition of re-yielding of a partially autofrettaged cylinder, we need only consider the stresses at the bore.

We now consider a cylinder having a diameter ratio exceeding 2.2 which would reverse yield if completely overstrained. The cylinder could be partially overstrained to an extent that the compressive tangential residual stress at the bore just equals the yield strength of the material. The pressure required to produce this condition is defined as the 100 percent autofrettage pressure, P_A, and is, of course, less than the complete overstrain pressure, P_c, for diameter ratios exceeding 2.2.

On a second application of P_A the stress state at the bore, assuming $\sigma_z = 0$, is:

$$\sigma_\theta = \sigma_{\theta e} \text{ due to } P_A + \sigma_\theta \text{ residual}$$

$$= P_A \left(\frac{K^2 + 1}{K^2 - 1} \right) - \sigma_y$$

$$\sigma_r = -P_A$$

Substituting these values into the Tresca yield criteria yields for P_A:

$$P_A = \sigma_y \frac{K^2 - 1}{K^2} \quad \ldots\ldots\ldots (57)$$

In a similar manner, the Maxwell-Mises yield equation gives for the 100% autofrettage pressure:

$$P_A = \sigma_y \left(\frac{3K^4 - 2K^2 - 1}{3K^4 + 1} \right) \quad \ldots\ldots\ldots (58)$$

Equations (57) and (58) give approximate values for the maximum pressure at which an autofrettaged cylinder can be designed to operate elastically. The assumption of no longitudinal stress in the development of this equation results in some error. However, in view of the small effect of longitudinal stress on yielding and overstrain pressures, it is assumed that this effect on P_A is also small.

Equation (58) is shown plotted in Fig. 3 where the maximum operating pressure for an autofrettaged cylinder can be compared with that for monobloc, multi-layer vessels, and the complete overstrain condition. Three points are important to note. First, eq. (58) predicts that the ratio of P_A to σ_y approaches unity as K becomes large. Thus, an autofrettaged pressure vessel cannot be designed to operate elastically at a pressure exceeding the yield strength of the material no matter how large the diameter ratio. Second, there is a considerable margin between the 100% autofrettage and complete overstrain pressure at large diameter ratios. Thus, as will be subsequently discussed, in some cases it is possible to operate at pressures in

excess of the 100% autofrettage pressure and approaching the complete overstrain pressure. Finally, one can, of course, by smaller amounts of overstrain, design for pressures lower than predicted by eq. (58).

e. Selection of degree of overstrain. In applying the autofrettage process to the design of an actual pressure vessel, a decision must be made concerning the amount of overstrain to be used. This decision must be based on a large number of factors and on considerable engineering judgment. Some of the factors which must be considered are: material properties such as strain hardening and ductility, safety factor desired, diameter ratio, autofrettage equipment available, number of pressure cycles expected, stress concentrations present, economic factors, importance of vessel weight and operating pressure. It is obvious that a complete analysis of all of these factors would be too lengthy to include herein. However, several of the more pertinent factors will now be considered.

For a material having limited strain hardening the maximum strength for elastic operation is obtained with 100% autofrettage. For the case of diameter ratios exceeding 2.2 this is obtained at less than complete overstrain and is generally the best design provided that pressure facilities are available for obtaining this degree of overstrain. Where vessel weight is a factor, such as in gun barrels, smaller diameter ratios are often used. In this case, the greatest strength is obtained with complete overstrain. However, for non-strain-hardening materials, this is difficult to obtain due to the risk of rupturing the cylinder since the complete overstrain pressure is also the rupture pressure. In practice this problem is overcome by using external restraining "containers" or dies to limit the enlargement obtained to

just exceed the complete overstrain condition. This method is well suited for large quantity production.

Referring to eq. (37) or eq. (54), we find that a cylinder autofrettaged so that 60 percent of the wall thickness is plastically deformed will have a strength only 6 percent less that that of a completely overstrained cylinder. This can be accomplished quite easily by using strain gages or other expansion measuring devices on the outside surface to monitor the strain during autofrettage.

If fatigue failure of the vessel is expected to be a problem, the degree of overstrain may be important particularly if the vessel must have stress concentration such as notches or holes at the outside surface. The high tensile residual stresses at this surface resulting from autofrettage combined with the tensile operating stresses can result in fatigue failures initiating at the outside surface. In this case there is probably some optimum amount of overstrain which will tend to balance the residual plus operating stresses at the two surfaces.

For a material which strain hardens appreciably, increases in strength can be obtained by exceeding the complete overstrain condition. However, such increases will be accompanied by appreciable decreases in the toughness of the material and should, therefore, be used with caution.

f. Methods of autofrettage. The autofrettage process involves the application of radial forces at the bore of a cylinder of sufficient magnitude to cause overstrain. These radial forces may be induced by either direct hydraulic pressure or by mechanical means. These two autofrettage techniques will now be described.

(1) Hydraulic autofrettage process. The principle of this autofrettage process and some of the process variables has been previously discussed. Therefore, little more need be said about this aspect.

There is nothing particularly complicated about the process. One has the choice of using an open-end process wherein the closures are supported externally by means of a press or an internal mandrel, or a closed-end technique where the closures are supported by the cylinder itself. The design theory for both cases has been presented and there is little difference in the end result. The open-end technique with the closures externally supported is generally the simplest and most economical approach since one, then, need not be concerned with highly stresses threads in the cylinder or a mandrel.

The conditions under which one must be concerned about the control of enlargement by means of external support have been previously discussed and will not be repeated. The design of containers for the control of enlargement during autofrettage is described in previous work by the present authors [26].

The hydraulic autofrettage process has the disadvantage that a means must be available for generating pressures equal to, or in excess of, the anticipated operating pressures. To overcome this

requirement, the mechanical autofrettage technique can be used to considerable advantage.

(2) <u>Mechanical autofrettage process</u>. This process, which has been described by Reiner and the present authors[27], and Manning[28], utilizes the mechanical advantage of a sliding conical wedge to produce the very high radial forces required for autofrettage. The method consists basically of forcing an oversized conical mandrel through the bore of the cylinder to produce the required radial expansion. The force required to move the mandrel may be supplied by some mechanical device such as a hydraulic press or by hydraulic pressure operating directly on the rear surface of the mandrel. An example of the latter method is illustrated in Fig. 6.

It has been shown[29] that the distribution of radial and tangential residual stresses produced by this method are effectively equal to that produced by conventional autofrettage. A significant longitudinal stress distribution is produced by this method but for reasonably small amounts of overstrain this has a negligible effect on the re-yield pressure.

In designing for the use of the mechanical autofrettage method, the general thick-walled cylinder theory, as previously presented, can be used. The required interference between the maximum diameter of the mandrel and the cylinder bore prior to autofrettage can be calculated as follows. The maximum internal pressure and the associated radial displacement of the bore at pressure required to produce the desired amount of overstrain is calculated using an appropriate theory for the

open-end condition. The initial interference is then equal to the maximum radial displacement of the cylinder plus the elastic radial contraction of the mandrel due to this maximum pressure. This can be calculated from the equation for the elastic contraction of a solid cylinder under radial pressure and no end load as follows:

$$\left(\frac{u}{r}\right)_m = \frac{P(1-\nu)}{E} \qquad \ldots\ldots (59)$$

where ν and E are the Poisson's ratio and elastic modulus of the mandrel material.

The final bore diameter after swaging is calculated using the Lamé equations and the above pressure to determine the elastic recovery from the maximum radial displacement as determined above.

The equations used in the above calculations were developed assuming a cylinder of infinite length. It would be expected that significant error would result from using these equations when the pressure is applied over only a short length of the cylinder at any time, as is the case in this method. However, considerable experimental data obtained both during process development[27] and actual production autofrettage by swaging indicate that this method of calculation is valid.

Although a complete discussion of the mechanical autofrettage process is given in Reference 27, some mention should be made of the problem of lubrication and the question of incremental enlargement. Due to the high interface pressures between the mandrel and cylinder

wall, friction is an important consideration. Although there has been no extensive investigation of lubricants, thin electro-deposited metallic layers, such as lead, have been found to be highly satisfactory.

The general practice is to obtain the total desired enlargement in a single pass of the mandrel through the bore, rather than by a series of mandrels of progressively increasing size. It is not likely that using the latter approach would offer any advantage since the radial forces for the last increment of enlargement would be effectively the same as that for the total simultaneous enlargement.

The mechanical autofrettage process offers economic and processing advantages of the hydraulic technique for certain cases. It permits the autofrettage of cylinders, at very high effective pressures, in the order of 15 to 20 kb; the only limiting feature being the compressive strength of the mandrel material.

g. <u>Effect of deviations from ideal material behavior on autofrettage theory and practice</u>. The preceding discussion of thick-walled cylinder theory, as indicated, was based on an assumed ideal, elastic-perfectly plastic material. All real engineering materials exhibit a number of deviations from this ideal material behavior. Three of these deviations which have the greatest effect on the elastic-plastic behavior of a thick-walled cylinder are (1) strain hardening, (2) Bauschinger effect and (3) strain aging.

(1) <u>Strain hardening</u>. Virtually all metals exhibit some degree of strain hardening or increase in flow stress with increasing plastic strain. This will affect both the elastic-plastic behavior of the cylinder and the behavior after complete overstrain has been reached including the ultimate strength or ductile rupture characteristics. However, many of the materials currently being used for very high pressure vessels, such as high strength 4340 steel or maraging steels, exhibit very little strain hardening. The fact that the diameter ratio of a thick-walled cylinder decreases slightly with increasing overstrain tends to counteract the effect of strain hardening. As an example, it was shown by the authors[26] that a 4340 steel cylinder, having a diameter ratio of 2.0 and a yield strength of 160,000 psi, would ultimately rupture if held at its complete overstrain pressure for sufficient time. Therefore, for this type of material the ultimate or rupture pressure is equal to the complete overstrain, or "collapse" pressure as given by eq. (52).

The effect of strain hardening on the elastic-plastic behavior has been studied by Liu[30] who published a solution of the partially plastic, thick-walled cylinder following the method of Allen and Sopwith[20] but considering strain hardening characteristics. He utilized a power strain hardening law of the form:

$$\sigma = \sigma_0 \epsilon^{\gamma}$$

where σ, ϵ and γ are the true stress, true strain and strain hardening exponent, respectively. This analysis only considers the problem up to the complete overstrain condition. Liu noted substantial deviations from those solutions assuming an elastic-perfectly plastic material, i.e., $\gamma = 0$, for values of γ exceeding 0.2. Since the strain hardening exponent for most high strength, engineering materials is considerably less than 0.2, the increased complexity introduced by considering strain hardening is generally not necessary within the partially plastic region.

If a material does exhibit appreciable strain hardening, the strength of the cylinder will continue to increase after the complete overstrain pressure is exceeded. Since the deformations associated with this large plastic flow are considerable the dimensional changes of the cylinder must be considered. This results in the fact that, as the pressure continues to increase, a point is reached at which the rate of strength increase due to strain hardening equals the rate of strength decrease due to the decrease in diameter ratio. The pressure at which

this occurs is called the ultimate pressure since beyond this point the cylinder is unstable and rupture will occur. This is analogous to the ultimate tensile strength and initiation of necking in a tensile test. Crossland et al[17] have shown experimentally that the ultimate pressure can be determined with reasonable accuracy for a closed end cylinder by:

$$P_u = 2\,\sigma_u \left(\frac{K - 1}{K + 1}\right) \quad \ldots\ldots (60)$$

for the case of a material of appreciable strain hardening and ductility.

It should be noted that the rupture pressures, as given in eq. (52) and particularly in eq. (60), apply to materials exhibiting considerable ductility and toughness. Many of the high strength materials used in very high pressure vessel design may not have this required ductility and thus will not be able to withstand the large bore strains associated with these equations. Thus, fracture can be expected at pressures less than the theoretical ultimate or rupture pressure. This point will be considered further in the section dealing with materials selection. It must be emphasized that the ultimate pressure should never be used as a basis of design but only as an indication of the upper limiting value that may be attained in a highly ductile material.

(2) Bauschinger effect. An effect, which is often neglected in thick-walled cylinder theory but can be very significant in certain cases, is

the Bauschinger effect. This phenomenon can be described as follows. A sample of a metal is subjected to a tensile load exceeding its yield strength resulting in a given amount of tensile plastic deformation. If this material is now loaded in compression, it will exhibit a compressive yield strength significantly less than its original yield strength. The extent of this effect is shown in Fig. 7 obtained by Milligan et al[31] which shows the compressive yield strength (at 0.2 percent offset) after tensile deformation versus the amount of tensile plastic flow for a 160,000 psi yield strength, 4330 steel.

The importance of this effect in an autofrettaged, thick-walled cylinder can be seen by examining the strain history of the material near the bore. During the application of the autofrettage pressure, this material undergoes tensile plastic flow ranging from 0.2 to 2.5 percent depending on the diameter ratio and amount of overstrain. From Fig. 7 we see that this will result in a decrease in compressive yield strength of 20 to 55 percent. As the pressure is decreased, the tangential stress decreases elastically to zero and then becomes compressive, approaching the theoretical residual compressive value. Due to the Bauschinger effect, however, this theoretical compressive stress may now exceed the reduced compressive yield strength. This will result in reverse yielding in cases for which no reverse yielding would be expected if the Bauschinger effect were neglected. The significance of this effect in large diameter ratio cylinders, and its relationship to the residual stresses and non-linearity in the pressure-strain curve upon unloading and reloading after overstrain, was pointed

out by Franklin and Morrison . In the lower diameter ratio range, recent results by the present authors indicated that reverse yielding will occur at a diameter ratio of approximately 1.8 instead of 2.2 as theoretically predicted.

As a cylinder which has reverse yielded as a result of the Bauschinger effect is re-pressurized to the original autofrettage pressure, the material near the bore will reyield in tension. When this pressure is released, this material will again reverse yield in compression. This phenomenon produces the so-called hysteresis loop effect in the pressure-strain curve which has been noted by many investigators.

The practical significance of this phenomenon is not generally understood. Since there are no appreciable permanent dimensional changes associated with this effect, for relatively few applications of pressure, it can be considered of no practical significance. However, there will certainly be some effect on fatigue behavior which will be discussed in another chapter. Also, the dilatation of the bore will be greater than that predicted by elastic theory which could cause seal or alignment problems.

The important thing to note is that, when the Bauschinger effect is considered, there is no longer any practical significance to the diameter ratio above which the classical theory predicts reverse yielding. The designer must accept the fact that some reverse yielding will occur in virtually any vessel operating at very high pressures.

The problem that the designer must solve is not to avoid reverse yielding but to decide how much can be tolerated. This will be based on the amount of recoverable plastic bore dilatation that is tolerable at maximum operating pressure and the fatigue life requirements. In fact, one can actually operate to the pressure corresponding to the complete overstrain condition as shown in Fig. 2 for diameter ratios exceeding 2.2 if some recoverable plastic deformation is not deemed deleterious to operational characteristics.

(3) Strain aging. The third type of deviation from ideal material behavior which should be considered is strain aging. This is a phenomenon which is seen in some materials, notably alloy steels, and can be described as follows. If the material is loaded in tension producing plastic deformation and then unloaded and heated at some specified temperature and time, a significant increase in tensile yield strength will result. For example, recent unpublished data by Milligan on 160,000 psi yield strength, 4330 steel shows an increase in yield strength of 15 percent after 1.5 percent plastic strain and heating to 600°F (315°C) for 3 hours.

It has been standard practice for many years in the autofrettage of gun tubes to subject the tubes to a thermal treatment after autofrettage. This has been found to decrease the hysteresis effect when the tube is re-pressurized. There are two possible explanations for this effect. The thermal treatment could either reduce the Bauschinger effect or, through strain aging, increase the yield strength

of the material. This thermal treatment is accomplished after the release of the autofrettage pressure and, therefore, after the reverse yielding caused by the Bauschinger effect has already occurred. It must then be assumed that the most significant effect of this thermal treatment is to increase the material yield strength through strain aging.

The practical benefit of this treatment is not fully understood. It has been shown by Davidson, Eisenstadt and Reiner[32] that this thermal treatment has no significant effect on the fatigue life of autofrettaged cylinders. This indicates that repeated pressure cycling tends to "wash out" the effects of this thermal treatment and/or the hysteresis effect for a non-thermal-treated cylinder.

3. Autofrettage and Multi-Layer Cylinders Combined

Based on purely theoretical considerations, the strongest possible cylinder design, without using a variable external restraint, can be obtained from an autofrettaged monobloc cylinder. However, there are several practical factors which impose limits on the use of autofrettage alone at very high pressures. As previously stated, the maximum reasonable design pressure for an autofrettaged cylinder is approximately equal to the yield strength of the material near the bore. At the time of this writing, the maximum strength materials available in a form suitable for vessel construction have an elastic strength in the order of 17 to 20 kb or somewhat higher if recoverable flow is permissible. We should then be able to construct autofrettaged vessels to operate in this pressure range. However, for operating pressures exceeding 10 kb the following practical factors must be considered.

1. The pressures required to autofrettage a vessel to operate in this pressure range may not be available from existing pressure generating equipment.

2. For large sized vessels metallurgical factors limit the size in which these high strength materials can be produced.

3. Since the very high strength is only required of the material near the bore, it is often uneconomical to use ultra-high strength material for the entire vessel.

4. Since these ultra-high strength materials have much greater tendency toward brittle fracture, the danger of catastrophic

failure of the vessel is greatly increased if it is constructed entirely of this material.

For the above reasons a design approach, which was previously proposed by the authors[5], is recommended for vessels operating at pressures exceeding 10 kb. This approach utilizes an ultra-high strength, autofrettaged liner combined with a lower strength jacket which may or may not be autofrettaged.

An example of this approach is shown in Fig. 8 which illustrates the design of a vessel having an elastic operating pressure of 245,000 psi (16.9 kb). This design utilizes a liner of 250,000 psi (17.2 kb) and a jacket of 160,000 psi (11.0 kb) yield strength respectively. For example, these could be a fully aged maraging steel liner and a tempered martensitic 4340 steel jacket. The liner is partially autofrettaged using a pressure of 190,000 psi (13.1 kb) which is within the capabilities of existing pressure generating equipment. This will produce the residual stress distribution shown in Fig. 8A based on the Tresca yield criterion. The jacket is now shrunk on so that the interface pressure is just sufficient to increase the compressive residual stress at the bore of the liner to equal the yield strength of the liner material (neglecting Bauschinger effect). The overall residual stress distribution after jacketing is shown in Fig. 8B and the total distribution of tangential and radial stress at the operating pressure is shown in Fig. 8C. This indicates that this vessel will operate elastically at this operating pressure. For some similar designs it will be found that the stresses at the bore of the jacket will exceed the yield

condition at the operating pressure. In this case, a small amount of autofrettage of the jacket prior to shrinking will eliminate this problem.

The stated values of material strength and hydrostatic pressure generating capabilities are consistent with current technology at the time of writing this report. It is anticipated that these values will steadily increase with advancing technology. Although the specific numbers may become outdated, the basic principles and design philosophy will remain valid.

C. Segmented Cylinders

The maximum operating pressure of the cylinder designs discussed thus far has been limited by the magnitude of the pressure-induced tangential stresses at the bore. However, if the cylinder is in some manner radially sectored, the thick-walled cylinder theory no longer applies and the stresses at the bore no longer represent a limiting factor.

The two general approaches for the application of the principle of segmented cylinders are shown schematically in Fig. 9 . The first of these wherein the intermediate cylinder is sectored, with the segments not being connected, will be considered in detail in a subsequent section. The second technique shown, which is termed the pin-segmented vessel, was originally proposed by Zeitlin et al[33], and subsequently investigated by Fiorentino et al[34]. In this case, the segments are not free, but are connected by pins. This design, however, does not offer any advantage with respect to maximum operating pressure over those previously discussed. It does offer the possibility of manufacturing vessels of extremely large size, i.e., up to 10 feet or more in outside diameter. Vessels of this size would not be feasible by other methods due to limitations in material fabrication and heat treatment.

D. Variable External Support

As previously stated, the use of residual stresses to increase the usable operating pressures of thick-walled cylinders is limited by the maximum compressive residual stress that the material can support when the internal pressure is effectively zero. This limitation can be overcome by some method which produces a compressive tangential stress at the bore of the cylinder that can be increased as the internal pressure is increased. This can be accomplished by subjecting the cylinder to an external pressure which is small when the internal pressure is zero and can be increased as the internal pressure is increased.

The effect of this external pressure can be seen if we examine the Lamé equation for the tangential bore stress due to both internal and external pressure. This is:

$$\sigma_{\theta r_1} = P_i \frac{K^2 + 1}{K^2 - 1} - P_c \frac{2K^2}{K^2 - 1} \qquad \ldots (61)$$

For a cylinder with a diameter ratio of 3.0, this equation becomes:

$$\sigma_{\theta r_1} = 1.25\ P_i - 2.25\ P_c \qquad \ldots . (62)$$

From this we see that the external pressure required to produce a given compressive tangential bore stress is considerably less than the internal pressure which would produce the same magnitude of tensile stress.

The various techniques utilizing a variable external support will now be considered.

1. Direct External Pressure

The obvious method of producing this variable external support is to place the cylinder within another pressure vessel or vessels and subject it directly to an external hydrostatic pressure which can be independently controlled. Such an approach was discussed by Bridgman[35]. More recently an approach based on the use of multi-layers was proposed by Berman[36] for pressures to 500 ksi. However, this method presents problems in generating, controlling and sealing this external pressure. Although these problems can be solved, there are other methods of accomplishing this same effect so that this method is not generally used. The analysis of this design to determine the maximum operating pressure is the same as that for the tapered external cylinder design which follows.

2. Tapered External Cylinder

A simple method for applying external restraint was developed by Bridgman[37]. This consists of making the pressure vessel with a tapered or conical outside surface. This is fitted into a heavy-walled jacket having a matching taper on the bore. As the pressure in the vessel is increased, the vessel is forced into the tapered jacket by a hydraulic press thus producing an external pressure on the vessel. A film of soft metal foil between the vessel and jacket provides sufficient lubrication. In this device the operating pressure in the vessel is generally developed by a piston moving directly within the vessel. The force on this ram is provided by a second hydraulic press. A schematic diagram of a typical design of this type is shown in Fig.10.

A detailed stress analysis of the inner cylinder of this device is difficult for the following reasons:

The maximum internal pressure is usually generated after the piston has moved a significant distance into the cylinder. The resulting elastic dilation of the inner cylinder causes a variation in the interface pressure along the length.

The variation in diameter ratio along the length results in further variations in interface pressure.

The pressure discontinuity at the point of seal of the piston causes bending stresses in addition to pressure stresses in the inner cylinder.

However, an approximate solution to this problem can be obtained if we neglect the above effects and consider a section at the

center of the pressurized portion of the vessel. The following assumptions are made in this analysis.

1. The yield strengths of the inner and outer cylinders are equal (σ_y).

2. The magnitude of the external pressure is limited by the strength of the jacket. Using an autofrettaged jacket, the maximum usable external pressure is given by eq. (57) if the diameter ratio of the jacket exceeds 2.22.

3. The liner is autofrettaged to produce a residual compressive tangential stress at the bore equal to the yield strength and has a diameter ratio exceeding 2.22.

4. The Tresca yield criterion is used and the longitudinal stress is assumed to be the intermediate stress.

The difference between the internal and external pressure at yielding of the inner cylinder based on eq. (57) is:

$$P_i - P_c = \sigma_{y1} \frac{K_1^2 - 1}{K_1^2} \quad \text{........} \quad (63)$$

For the condition of simultaneous yielding of both cylinders, P_c is the yield pressure for the outer cylinder which is:

$$P_o = \sigma_{y2} \frac{K_2^2 - 1}{K_2^2} \quad \text{........} \quad (64)$$

Substituting eq. (64) into eq. (63) and determining the value of K_2 which provides the maximum value of P_i for a given K_t, as in the case of the multi-layer vessel, yields:

$$K_2^2 = K_1^2 = K_t \qquad \qquad . . . . (65)$$

if $\sigma_{y1} = \sigma_{y2}$.

The maximum yield pressure then is given by:

$$\frac{P_Y}{\sigma_y} = 2 \left(\frac{K_t - 1}{K_t}\right) \qquad \qquad . . . . (66)$$

This, then, gives an approximate upper limit design pressure for this type of vessel.

In order that the fourth assumption be satisfied, a high compressive longitudinal stress must exist in the inner cylinder at the operating pressure. In the tapered jacket design, this is provided by the hydraulic press which forces the inner cylinder into the tapered jacket. Since this longitudinal stress approaches zero at the small end of the jacket, the maximum operating pressure should not be permitted to exist near this end of the vessel.

From the above analysis, it can be seen that the maximum operating pressure for a vessel utilizing external support is limited by the maximum external pressure that can be applied to the inner cylinder. This is true regardless of the method which is used to apply this restraint. If the external pressure is provided either by a tapered jacket or by direct hydrostatic pressure, this external pressure is limited to the strength of the jacket or outer vessel. However, if some method is used which is not subject to the limitations of thick-walled cylinder theory, it is possible to apply significantly higher external pressures and thus obtain higher operating pressures. Such an approach will be described in section 3d.

3. Segmented Cylinder

The concept of a segmented cylinder, the principle of which has been previously discussed, is not limited to use as a means of providing variable external support for a load carrying inner cylinder. It can also be used to contain the internal pressure directly with a thin liner to prevent leakage of fluid into the spaces between the segments. In this context it is not appropriately considered a case of variable external support, as are the two previous designs, and would more appropriately have been discussed in the previous section on segmented cylinders. However, due to the similarity of the design analysis used, all types of vessel designs utilizing segmented cylinders will be considered in this section.

As indicated above, the segmented cylinder can be utilized in a number of different ways in pressure vessel designs. Since a segmented cylinder cannot in itself support any internal pressure, its use is based on the application of an external force on the segments which intensify and transmit this force to an inner cylinder that actually contains the pressure. The external force may be applied by means of direct mechanical contact with an outer cylinder or by hydraulic pressure applied to the outside surface of the segmented cylinder and contained within an external cylinder. The internal cylinder which is supported by the segments can be either a thin liner which is not an actual structural member but simply prevents the pressure fluid from entering the space between the segments, or a thick cylinder which forms a structural component of the pressure vessel. In considering

the design analysis then, the various applications of a segmented cylinder can be divided into (a) thin liner - solid outer cylinder, (b) thick-walled liner - solid outer cylinder, (c) thin liner - variable external pressure, and (d) thick-walled liner - variable external pressure. These various configurations will be separately discussed.

a. <u>Thin liner - solid outer cylinder</u>. This configuration, which is shown schematically in Fig. 9, was originally proposed by Poulter[38] and has been further investigated by Pugh[6,39] for the case of a non-autofrettaged, monobloc outer cylinder. Pugh's analysis, which will now be presented, is based on the assumption that simultaneous yielding of the segments and outer cylinder represents the optimum condition.

At the maximum operating pressure, the segments are assumed not to be in contact with each other and thus, from a force balance on the segments, the ratio of internal (P_i) to interface (P_{12}) pressure between the segments and outer cylinder is:

$$P_i = K_1 P_{12} \qquad \ldots\ldots\ldots\ldots (67)$$

where K_1 is the diameter ratio of the segmented cylinder.

The condition of yielding at the bore of the outer cylinder using the Tresca yield criterion is given by eq. (9). Substituting this for P_{12} in eq. (67) yields:

$$P_y = \frac{\sigma_{y2} K_t}{2} \left(\frac{1}{K_2} - \frac{1}{K_2^3}\right) \qquad \ldots\ldots (68)$$

Differentiating eq. (68) and equating to zero yields the value of K_2 which will produce the maximum value of P_y for any given K_t. This yields

$$K_{2\ opt.} = \sqrt{3} \qquad \ldots\ldots (69)$$

Pugh gives the condition for yielding of the segments as:

$$P_y = \sigma_{y1} (1 + \alpha) \qquad \ldots\ldots (70)$$

where α is the semi-angle of the segments and normally has values of $\frac{\pi}{6}$ or $\frac{\pi}{8}$. Equating eqs. (68) and (70) and substituting the optimum K_2 from eq. (69) gives the following condition for simultaneous yielding at both bore surfaces.

$$K_{t\ opt.} = \frac{\sigma_{y1}}{\sigma_{y2}} (1 + \alpha)\ 3\sqrt{3} \qquad \ldots\ldots (71)$$

The maximum operating pressure, for diameter ratios less than optimum, i.e., $K_t < K_{t\ opt.}$, is obtained by the substitution of eq. (69) into (68) as follows:

$$P_y = \frac{\sigma_{y2}\ K_t}{3\sqrt{3}} \qquad \ldots\ldots\ldots (72)$$

For a diameter ratio equal or greater than $K_{t\ opt.}$ yielding of the segments is the controlling factor and the maximum P_y is given by eq. (70).

Assuming equal strength of the segments and outer cylinder, based on the consideration of stress alone, this type of segmented vessel design offers considerable advantage in terms of maximum operating pressure over the multi-layer and autofrettaged designs considered thus far. However in order to fully utilize the potential of the segmented cylinder design, it is advantageous to use a material of higher compressive strength for the segmented cylinder. As can be seen from eq. (71), this will result in very large diameter ratios for an optimum design, viz. $K_t = 15.6$ if $\sigma_{y1} = 2\ \sigma_{y2}$. This diameter ratio can be reduced by utilizing an autofrettaged outer cylinder as will now be discussed.

The maximum elastic operating pressure for an autofrettaged cylinder is given by either eq. (38) or eq. (57) based on the Tresca yield criterion. The condition for yielding of the outer cylinder then becomes:

$$P_y = \frac{\sigma_{y2} K_t}{K_2} \ln K_2, \quad K_2 \leq 2.22 \qquad \ldots\ldots (73)$$

or

$$P_y = \sigma_{y2} K_t \left(\frac{1}{K_2} - \frac{1}{K_2^3} \right), \quad K_2 \geq 2.22 \qquad \ldots\ldots (74)$$

From eqs. (73) and (74) the optimum value of K_2 is found to be 2.22.

Since in actual practice one would generally use a value of K_2 slightly greater than 2.22, the condition for simultaneous yielding from eqs. (74) and (70) is

$$K_{t\ opt.} = \frac{\sigma_{y1}}{\sigma_{y2}} (1 + \alpha) \left(\frac{K_2^3}{K_2^2 - 1} \right) \qquad \ldots\ldots (75)$$

For the optimum value of K_2 this becomes

$$K_{t\ opt.} = \frac{\sigma_{y1}}{\sigma_{y2}} (1 + \alpha) (2.785) \qquad \ldots\ldots (76)$$

which gives a substantially lower value of the optimum diameter ratio. Since both designs are limited by yielding of the segments, they both have the same maximum operating pressure for the same segment material as given by eq. (70).

The above analysis has considered only the stresses. It has been noted by Fiorentino, et al[34] and by Pugh[35] that, if the segmented

cylinder and the outer cylinder are assembled without significant interference and compressive pre-stress in the segments, the dilations at the bore of the segmented cylinder will be large. This will result in large plastic strains in the inner liner and possibly extrusion of the liner material into the spaces between the segments. As suggested by Pugh, this problem can be partly overcome by assembling the cylinders with an interference fit resulting in compressive tangential stresses in the segmented cylinder. As the internal pressure is increased, the segmented cylinder will act as a complete cylinder as long as the tangential stresses remain compressive. This will result in significantly smaller dilations at the bore of the segmented cylinder. However, the advantage of a segmented cylinder is its ability to transmit and intensify an external pressure as [illegible] in eq. (67). This requires that the segments not be in contact at the maximum pressure. The optimum design would, therefore, be one in which the initial shrinkage interface pressure is such that yielding will occur at both the bore of the outer cylinder and the segments at the pressure which will just produce complete separation of the segments.

If the segments are machined so that the contacting surfaces are parallel in the un-stressed condition as would be required in order to apply the Lamé equations to the segmented cylinder, the complete separation of the segments does not occur at one value of internal pressure but occurs first at the bore and progresses outward with increasing internal pressure. The analytical solution for this design between the point of initial and final separation is difficult since

it becomes a statically indeterminate structures problem and the Lamé equations no longer apply to the segmented cylinder. There is no known solution to this problem.

In order to obtain an upper limit to the optimum value of initial shrinkage interface pressure, it can be seen that it must be less than the value which would result in simultaneous yielding of the outer cylinder and initial separation of the segments at the bore. The solution for this case can be obtained since the segmented cylinder can be considered as a continuous cylinder and the Lamé equations used. This solution will now be presented.

The Lamé equations for combined internal and external pressure and the condition of separation of the segments at the bore (i.e., $\sigma_{\theta@r} = r_1 = 0$) yields:

$$P_i^* (1 + K_1^2) = 2K_1^2 P_{12}^* \qquad \ldots\ldots (77)$$

It should be noted that P_i^* and P_{12}^* apply to the case of simultaneous separation of the segments and yielding of the outer cylinder. Thus, they are not the same as P_i and P_{12}, as given in eq. (67) which are based on the case of complete separation of the segments. Thus, the previously given relationships involving P_i and P_{12} are used to determine vessel operation pressure and optimum configuration. P_i^* and P_{12}^* apply only to the question of the optimum amount of initial interference between the segments and the outer cylinder.

The value of P_{12}^* which will yield the outer cylinder, using an autofrettaged cylinder of the optimum diameter ratio (i.e., $K_2 = 2.22$), is:

$$P_{12}^* = .797\ \sigma_{y2} \qquad \ldots\ldots\ldots (78)$$

Substituting eq. (78) into eq. (77) yields:

$$P_i^* = \left(\frac{2 K_1^2}{1 + K_1^2} \right) .797 \sigma_{y2} \quad \ldots\ldots \quad (79)$$

Equations (78) and (79) give the pressures existing at the bore of both cylinders at the condition of simultaneous yielding of the outer cylinder and separation of the segments.

In order to design a vessel, it is necessary to know the value of initial shrinkage interface pressure, P_{12}^I, which will produce the above condition. To obtain this initial value, a general relationship between P_{12} and P_i is required which will apply as long as the segments remain in contact. This relationship is obtained by equating the elastic strain at the bore of the outer cylinder due to P_{12} to the elastic strain at the outside surface of the segmented cylinder due to P_{12} and P_i. The relationships for these strains are obtained from the Lamé equations and Hooke's law, resulting in the following:

$$\frac{2P_i - P_{12} \left[K_1^2 (1 - \nu_1) + (1 + \nu_1)\right]}{E_1 (K_1^2 - 1)} = \frac{P_{12} \left[K_2^2 (1 + \nu_2) + (1 - \nu_2)\right]}{E_2 (K_2^2 - 1)} \quad (80)$$

The initial interface pressure, P_{12}^I, can now be determined by calculating the change in P_{12} resulting from changing P_i from the value at pressure (P_i^*) to zero. The absolute value of this change in P_{12} is found by substituting P_i^* from eq. (79) into eq. (80) which yields ΔP_{12}. The initial interface pressure is then given by:

$$P_{12}^I = P_{12}^* - \Delta P_{12} \quad \ldots\ldots\ldots \quad (81)$$

where P_{12}^{*} is given by eq. (78).

The radial interference between the bore of the outer cylinder and the outside diameter of the segmented cylinder prior to assembly can be determined from eq. (24) given in the section on multi-layer cylinders.

Two points are important to note with respect to the behavior of the thin liner and its effect upon the maximum operating pressure. First, due to the high radial and tangential stresses in the liner at high pressure, substantial plastic flow and extrusion in the longitudinal direction may occur if no means are utilized to control the longitudinal stresses. Thus, a system to provide a variable longitudinal compressive stress in the liner must be provided. The friction between the liner and the segments may be sufficient to provide this stress.

The second point involves the radial displacement of the liner. Although the separation of the segments with possible failure of the liner due to excessive strain and/or extrusion between the segments can be minimized by the use of a pre-stressed jacket, or possibly by pre-shaping the segments to retard initial separation, even further steps may be required. A possible approach in which this problem may be eliminated is to use a variable external pressure between the segments and outer cylinder, as will be subsequently discussed.

b. <u>Thick-walled liner - solid outer cylinder</u>. As previously discussed, the pressure which a thick-walled cylinder can contain can be greatly increased by subjecting the cylinder to an external pressure. It would therefore appear that if one utilized the very high pressure containing capability of a segmented cylinder to apply an external pressure, extremely high internal pressures could be contained. However, as discussed in the section on thin liners, the dilation of the bore of the segmented cylinders at the maximum pressure may be sufficiently large to permit plastic strains in the inner cylinder. Therefore, on the release of the maximum pressure, reverse plastic strains will occur in the liner similar to reverse yielding in an autofrettaged cylinder. Since the operating limitations of this design are based on the values of plastic strains which can be tolerated and since there is no known solution for the magnitude of these strains, no specific maximum operating pressure can be given. It is probable, however, that this design offers no significant advantage over the thin liner design.

c. Thin liner - variable external press: The use of a variable external pressure segmented vessel was investigated by Ballhausen[40], Gerard and Brayman[41], Fuchs[42], Levey and Huddleston[43]. It is worthy to note that this approach has also found considerable application in apparatus utilizing solid pressure transmitting media, as well as in the hydrostatic type pressure vessel of concern in this chapter.

As previously discussed, the use of a segmented cylinder supported directly by an outer cylinder is theoretically limited by the strength of the segments. Thus, there appears to be no advantage to be gained strengthwise by supporting the segmented cylinder by a variable external pressure. However, the practical limitation of the previously discussed designs may not be strength alone but the amount of dilation which can be tolerated at the bore of the segmented cylinder. This dilation can be minimized by supporting the segmented cylinder by a variable external pressure which is increased as some predetermined function of the internal pressure. This is accomplished by placing a thin jacket around the outside surface of the segmented cylinder to prevent leakage of pressure between the segments.

The optimum condition would be to have the magnitude of the external pressure such that, for all values of internal pressure, the segments would be just contacting but with no compressive tangential stresses. This would provide the desired pressure intensification and a minimum dilation at the bore of the segmented cylinder. However, due

to elastic deformations of the segments, as studied by Fiorentino et al[34] this condition cannot be precisely obtained. However as pointed out by Fiorentino, by proper initial shaping of the segments a reasonable approximation of this condition can be achieved.

d. Thick-walled liner - variable external pressure. A logical extension of the externally pressurized segmented cylinder concept is to use it as a means of applying a variable external support to a load-bearing or thick-walled inner liner. An example of such an approach is shown schematically in Fig. 11.

The analysis of the inner cylinder of this design is essentially the same as that for the tapered external cylinder design except that the external constraint pressure P_o, as given in eq. (64), may now exceed σ_y and approach the compressive strength of the segments as given in eq. (70). Thus, incorporating eq. (70) into eq. (63) gives for the operating pressure:

$$P_i = \frac{\sigma_{y1} (K_1^2 - 1)}{K_1^2} + (1 + \alpha) \sigma_{y2} \quad \quad . \; . \; . \; . \; (82)$$

This analysis again assumes that the longitudinal stress in the inner cylinder is the intermediate principal stress. Therefore, to use this design to maximum advantage, it is necessary to apply a variable compressive longitudinal stress to the inner liner. This can be accomplished in a variety of ways, including the use of a separate external jack, or by using the variable external pressure as shown schematically in Fig. 11. Another approach is that used by Fuchs[42] wherein by using leakage paths, the internal pressure automatically

supplies and controls the external pressure and, also, the end loading of the inner liner.

From a practical standpoint it is important to note that the limiting factor in this design may no longer be the strength of the cylinder but the compressive strength of the piston used to generate the pressure.

E. Summary of Pressure Vessel Design

The theoretical approaches to a variety of pressure vessel designs have been discussed. As previously stated, the type and construction of the vessel one chooses depends upon the experiment and may range from a simple monobloc to a complex arrangement utilizing several of the principles described. The decision as to which design to use depends to a great extent upon the pressure level desired and materials available. As a simplified guide in such a selection, Table I shows a comparison of the maximum theoretical elastic pressure limits in terms of the yield strength of the materials along with the pertinent design relationship for the various configurations previously described. These relationships are shown plotted in Fig. 12. It should be noted that, for comparison purposes, specific assumptions, as shown in Table I, are made concerning some of the variables in several of the design concepts. For simplicity, the values and relationships shown are based on the Tresca yield criterion assuming that the longitudinal stress is intermediate. The relationships given then represent only a guide and the actual design of a vessel must take into account all possible theories and variables and those best fitting the actual condition chosen. It is also important to consider that the elastic solutions presented along with the pressure limits shown in Table I are eloped assuming an infinite length cylinder. In practice, the vessel itself is not of infinite length and the pressure, in some instances, is exerted over only a short length of the vessel. The effect of this will tend to make the upper pressure limit values shown

somewhat conservative due to the restraining effects of the unpressurized length of the cylinder.

There are several points of interest with respect to Table I and Fig. 12. First, in the limit of diameter ratio, the elastic pressure limit for the multi-layer case is equivalent to the autofrettaged condition. This needs some clarification in view of previous statements. Referring to Fig. 3 it can be seen that the autofrettaged case approaches σ_y much more rapidly than does the multi-layer case. However, as shown by comparison of the curves for the two and three element multi-layer cylinders, as the number of elements increases, the multi-layer curve more closely approaches that for the autofrettaged case at all diameter ratios.

Due to reverse yielding the 100% autofrettaged curve, which corresponds to the maximum elastic operating pressure, differs from the complete overstrain curve for cylinders having diameter ratios exceeding approximately 2.2. In reality an autofrettaged vessel of a diameter ratio greater than 2.2 can be autofrettaged and operated at pressures exceeding the yield strength and as high as the complete overstrain pressure if recoverable plastic flow is permissible and the ductility of the material is sufficient to withstand the strains involved. In contrast, if the maximum elastic pressure in the multi-layer vessel is exceeded, the vessel will undergo irreversible plastic flow.

As shown, the use of a variable external support, whether by means of a direct external pressure or a tapered external cylinder, significantly extends the elastic operating pressure over that obtained

by autofrettaged or multi-layer construction. It should be noted, however, that autofrettage of the liner and outer cylinder is necessary to achieve the elastic operating pressures shown.

The highest permissible elastic operating pressures are obtained by those configurations based on the use of a segmented, or sectored, cylinder. As shown in Table I, the relationship and the maximum operating pressure, as shown in the second curve from the top of Fig. 12, for the thin and thick-walled liner - solid outer cylinder and the thin liner - variable external pressure segmented vessel designs are the same. The only advantage, in this case, of using a variable outer pressure, as compared to the solid autofrettaged outer cylinder, is to reduce the problem of excessive strains and perhaps extrusion of the liner.

Using a segmented vessel with variable external pressure as a means of variable support for a thick-liner has the potential of achieving the highest operating pressure of those designs considered. The increased elastic operating pressure of this over the thin-liner segmented vessel design is due to the strength contribution of the thick-walled autofrettaged liner.

Some mention should be made of the form of the curves shown in Fig. 12 for the segmented vessel designs. First, due to the assumptions of minimum diameter ratios, as listed in Table I, the curves do not extend from $K_t = 1$. Second, the curves are discontinuous and effectively level off when the condition of yielding in the segments becomes controlling. This condition is defined by the second equation

in Table I for each of the segmented vessel designs where it has been assumed that $\alpha = \frac{\pi}{6}$.

In closing this discussion of pressure vessel design, attention should be drawn to two quite recent papers by Wilson and Skelton[44] and Babb[45] which have not been previously referenced. Both papers contain excellent reviews of many aspects of pressure vessel design. The latter paper also considers a number of the practical problems associated with the choice, fabrication and utilization of pressure vessels and high pressure equipment.

III. MATERIALS FOR PRESSURE VESSELS

The choice of materials for pressure vessels depends upon several factors including stress level, stress state, number of cycles, construction, stress discontinuities, failure criterion, environment, etc. Here again, as in the case of vessel design, it seems more pertinent in view of the wide spectrum of vessel design and application to consider the principles of material selection rather than specific materials for each possible application.

To start, it may be helpful to consider what properties of a material are significant in pressure vessel applications. In some instances, the significance of the property is obvious. In other cases however, it is not so obvious and has often either been overlooked or neglected resulting in some failures.

A. Yield Strength

Yield strength is one of those properties whose significance is obvious and little need be said. One point worthy of mention, however, is that, in some materials from a practical standpoint, yield strength, or more appropriately fracture strength in the case of highly brittle materials, is anisotropic. For example, in some high carbon tool steels, carbide, etc., the strength in compression far exceeds the strength in tension. In the use of such materials then, one must consider the difference in the ability to withstand compressive versus tensile stresses. Although usually such materials are only used where the design stresses are principally compressive, one must insure that, during the pressure cycle, they do not become significantly tensile due to the pressure itself, or the presence of bending or stress discontinuities or stress concentrations. For example, tungsten carbide is a highly desirable material for the segments of segmented vessels since the principal stress is compressive with no appreciable tensile tangential stresses due to the discontinuous sections.

B. Ductility and Toughness

All too often ductility and toughness have not been considered in the selection of a material. There has been a tendency to rationalize that if one can use a material of sufficient strength so that no plastic deformation will occur, ductility and toughness are of no importance. This often leads to the use of very high strength but often highly brittle materials. There are several pitfalls in the above rationalization. One may not be able to accurately predict the actual stress level. This may be due to the inherent inaccuracy of the design theory or the presence of stress discontinuities, and stress concentrations. Thus, in any vessel, one might readily encounter either gross or localized stresses that exceed the yield strength. Thus, if the material has little or no ability to accommodate plastic flow without fracture, one will be faced with the potential of a catastrophic failure. Another consideration is that if autofrettage is going to be used, then the material must have sufficient ductility to withstand the plastic flow during autofrettage.

Toughness, or more specifically fracture toughness, is a measure of the ability of a material to resist the propagation of a crack. As in the case of ductility, toughness generally decreases and thus becomes more significant with increasing strength level. Fracture toughness is an important consideration from several standpoints. First, as fracture toughness decreases, the size of a crack or defect that can be tolerated at a given stress level decreases according to the highly simplified relationship:

$$K_{Ic} = c\sigma a^{\frac{1}{2}} \quad \ldots\ldots\ldots (83)$$

where K_{Ic} is the plane strain fracture toughness, σ the applied stress, "a" the critical crack size (depth in the case of a cylinder) and "c" a constant. One reason for neglect of fracture toughness in pressure vessel material selection is the obvious argument that no cracks are present nor would be tolerated. However, one must consider that, in some cases, very high strength materials will fracture before the anticipated yield strength even though no observable initial cracks are present. This can be attributed to the fact that the critical crack size is so small that inherent material defects and discontinuities are sufficient to exceed the critical size prior to the yield stress. In addition, during operation surface cracks or discontinuities may be introduced due to the effects of metallic seals, fatigue, or damaging environments. Obviously, it would not be desirable to have the condition where this surface damage is sufficient to exceed the critical crack size for the operating stress level.

Fracture toughness is also quite important from the standpoint of fatigue since the depth to which a fatigue crack can propagate before becoming a fast or running crack is directly related to the fracture toughness as shown in eq. (83).

Some mention should be made as to the measurement of toughness. The most commonly used "measure" is an impact test using either the charpy or Izod specimen configuration. This is not a fracture toughness test per se since one is measuring both the energy for initiation

and propagation. It is, therefore, more of a qualitative comparison type test yielding little, if any, quantitative information, particularly at very high strength-low toughness levels. The actual measurement of fracture toughness involves the use of a pre-cracked specimen subsequently loaded in tension or bending with the load required to cause a crack of a given size to propagate being the basis for the computation of fracture toughness. Such measurements are becoming more common and data is available for a large number of materials. Thus, the use of fracture toughness data should be considered for use rather than the impact type data.

C. Available Materials

It may be helpful to consider briefly some of those materials either commonly used or having potential for use in pressure vessels. It should be realized, of course, that each builder or user of pressure vessels has particular materials that they favor, perhaps in some instances because they have worked in the past. If a particular material is neglected in the following discussion, it is not because it is considered unsatisfactory, but only due to that fact that consideration of all materials or combinations would be unnecessary.

Up to the strength level of approximately 180 ksi, the AISI 4140 and 4340 classes of materials (see Table II for typical chemical compositions) have been widely and successfully used for a long period of time. By low temperature tempering, the strengths of these materials have been extended to over 200 ksi. However, one pays a penalty in toughness and ductility as compared to other newer alloys.

In the range of 200-250 ksi yield strength, one has several choices of materials. The AISI S-5 and S-7 impact resisting tool steels have been widely and successfully used. However, even a higher ductility and toughness in this strength range can be obtained in two new families of materials consisting of the HP nickel-cobalt quenched and tempered carbon steels and the 18% nickel maraging steels. Both of the latter materials show considerable promise for pressure vessel applications.

Above 250 ksi yield strength, the number of materials available for pressure vessel applications is quite small. An 18% Ni

maraging steel having a 280 ksi strength level is available, but due to processing problems with this particular alloy, there is a significant drop in ductility and toughness in large section sizes as compared to the 250 grade. New grades of maraging steels with yield strengths up to 500 ksi are currently under development.

Some special tool steels having yield strengths somewhat in excess of 300 ksi are available. However, these materials by their very nature and strength level have quite low ductility and toughness. For pressure vessel applications involving high tensile stresses, they must be used with extreme caution in order to reduce the probability of brittle catastrophic fracture.

Many materials considered unsatisfactory for use in vessels wherein high tensile stresses are inherent are highly adaptable to cases such as the segments in the segmented vessel design, closures and pistons where the principal stress is compressive. Compressive yield strengths somewhat above 400 ksi can be obtained with several tool steels and specialty steels of .8 to 1.0% carbon content as typified by HTB-2 (Allegheny) and Versatool (Crucible).

For elevated temperature ($> 1000^{\circ}F$, $540^{\circ}C$) applications the nickel based "superalloys", such as René 41, Waspaloy and Inconel 718, are suitable.

The material generally used for applications involving compressive stresses in excess of 400 ksi is cemented carbide. Compressive strengths of as high as 600-800 ksi are obtainable with tungsten

carbides having a cobalt binder. The compressive strengths vary immensely as a function of the cobalt binder content ranging from 400-500 with 25% to 500-800 ksi with 3% cobalt respectively. The ductility, toughness, and transverse rupture strength increase with increasing cobalt content, which is an important consideration if the possibility of stress discontinuities or small tensile stresses exist.

D. Environmental Factors

Aside from the obvious environment of high pressures, a vessel may also be subjected to other factors such as high or low temperatures and corrosive or reactive media.

Low temperatures may result in significant reductions in toughness of the vessel material. It is important, therefore, to apply the preceding discussion of toughness in terms of the properties of the material at the lowest operating temperature expected.

For vessels operating at elevated temperatures the decrease in yield strength with increasing temperature must be considered. Also, if the vessel is to be subjected to high pressure and temperature for any appreciable time, the phenomenon of creep must be considered. Under these conditions the criterion of failure will be stress rupture, and the vessel design must be based on a compromise between maximum operating pressure and allowable time at pressure for a given temperature. Creep and stress rupture data are readily available in the literature or from the suppliers of high temperature alloys.

It is obvious that the use of corrosive media in high pressure vessels should be avoided. However, when corrosive media or media which can result in hydrogen embrittlement must be utilized, the problem of stress corrosion cracking must be considered. Since space limitations of this report do not permit a complete discussion of this problem, this section is intended to serve only as a warning that this problem exists and must be considered in vessel design. It should be

pointed out that even such seemingly inert environments as distilled water may produce serious stress corrosion problems under certain conditions.

IV. SEALS, PISTONS AND CLOSURES

The sealing of very high pressures is often surprisingly easy. Since the seal can usually be designed so that the pressure tends to force the sealing surfaces together, then the higher the pressure the tighter the seal.

The simplest form of a high pressure seal utilizes a pre-compressed soft material such as rubber or plastic. This can vary from a simple rubber washer, or so-called "Poulter" packing, to a complicated multi-lip composite material seal. However, since these various seal designs are generally limited to pressures less than 0.5 kb and are discussed at considerable length in a number of standard references[46-49] they will not be discussed in detail herein.

Although the above seals are designed for use at low pressures, they can be used at much higher pressures provided that the seal is completely contained and that any clearances are small enough to prevent extrusion of the soft material. However, due to the considerable elastic deformations of pressure vessel components, these small clearances cannot generally be maintained at very high pressures resulting in extrusion and failure of the seal.

Most very high pressure seals use a metal-to-metal contact surface either alone, where it acts as both an initial and a final seal, or in combination with one or more softer deformable materials to provide the initial seal. An exception to the use of a metal-to-metal seal is the case where electrical insulation is required such as in electrical leads passing into the pressure environment. Such insulated seals will be discussed later.

The use of metal-to-metal seals without a separate means of obtaining an initial seal involves some special considerations which limit their use. Since two metal surfaces in elastic contact will usually provide some fluid leakage path due to surface irregularities, obtaining an initial seal can be a problem. This can sometimes be overcome by mechanically forcing the sealing surfaces together, thus plastically deforming one or both surfaces to obtain complete initial contact. One must be careful, however, to insure that elastic deformations resulting from the pressure do not relieve the initial preload force thus permitting leakage at high pressures.

An example of a metal-to-metal seal without a separate means of obtaining an initial seal is the cone seal which is commonly used as a high pressure tubing connection. This is shown in Fig.13A and consists simply of forcing a conical member containing a small concentric hole into a small conical seat in the fitting or vessel to which the connection is being made. This conical member may be simply a conical end on the thick-walled tubing itself or it may be a short cylinder with two conical ends. This is placed between the end of the tubing and the fitting or vessel each of which contains a mating seat. This type of seal is extensively used for tubing connections at pressures up to about 14 kb. An included cone angle of 60° is generally used with the angle of the cone slightly smaller than that of the seat to assure that initial contact will occur adjacent to the pressurized region.

A variation on the double ended cone design is the "lens ring" seal shown in Fig. 13B . This consists of a short, double ended

cone having a large included angle, usually in the order of 150°. This design is useful for sealing slightly larger diameters than the cone seal. The mating part to the lens ring is made with a square ended hole slightly larger than the hole through the lens ring. A large make-up force is used to deform the material adjacent to the hole producing an initial seal. As the pressure is increased, the radial dilation of the lens ring due to its internal pressure produces a wedging action which tends to increase the contact pressure between the lens ring and its mating surfaces. The principal problem with this design is the need for very accurate alignment of the two components being connected and a very rigid mechanical connection.

Most seals for very high pressure applications are of the combination type, i.e., a metal-to-metal final seal with a soft material such as an elastomer to provide an initial seal.

Depending upon their application, high pressure seals can generally be divided into static and dynamic types. A dynamic seal is one which provides a seal between two surfaces which move relative to each other. This can be either the case in which the seal itself moves, such as on a moving piston, or in which the seal itself is stationary and seals against a moving surface, such as a piston rod "gland" seal. A static seal is, of course, one in which there is no relative movement between the sealing surfaces. It should be noted that some relative motion between the sealing surfaces is virtually always present due to elastic deformations of the vessel components. These deformations

should be considered to ensure that they will not result in seal failure due to excessive clearances or loss of initial seal.

Since most of the seals to be discussed can be adapted to either static or dynamic applications, the following discussion will not be divided in this manner.

The two most commonly used seals for very high pressure applications are the unsupported area or "Bridgman" seal and the wedge ring seal. These are shown in Fig. 14. Actually both of these designs utilize the principal of an unsupported area. The fact that a portion of the seal on the side away from the pressure is not completely supported results in an intensification of the pressure within the seal material. Thus, the seal exerts a pressure on the cylinder wall greater than the pressure being contained which is necessary to prevent leakage. The "Bridgman" design is most generally used as a dynamic seal on high pressure pistons. The principal problem with this design is the fact that the part designated as the socket in Fig. 14A must be capable of supporting a longitudinal stress exceeding the operating pressure. This often requires the use of very hard, brittle materials and thus care must be taken to ensure that no stress concentrations or bending stresses are present. The existence of a pressure exceeding the operating pressure in the sealing rings may also cause "pinch off" failure of the stem due to the radial compressive stresses. However, through careful design and selection of materials these problems can be overcome and this design is very successfully used for pistons operating at pressures up to 30 kb.

The wedge ring design is shown in Fig. 14B and is widely used in static seals such as vessel end closures and in dynamic seals such as piston rod gland seals. The unsupported area in this case is the area of the lower surface of the wedge ring. The amount of this unsupported area can be varied from the full projected area of the ring (square ring) to the area of the clearance between the closure and the cylinder (triangular ring).

The use of a square cross-section ring in combination with a series of pre-compressed washers to provide the initial seal was proposed and extensively used by Bridgman[50]. The square ring in combination with an "O" ring for initial seal has been widely utilized in a number of applications including the autofrettage of large gun tubes[26]. Such a design is quite useful in applications involving one, or a few pressure applications and where surface finishes and tolerances are not closely controlled.

The triangular cross-section wedge ring has been proposed by Newhall[51] in conjunction with a "U" type packing to provide an initial seal, and subsequently utilized in combination with a variety of techniques for initial sealing. Due to the small unsupported area, better surface finishes are required to ensure that the initial seal is maintained. This seal can also be used as a dynamic seal. However, its life is limited by wearing away of the relatively thin wedge ring. It can be used in static applications repeatedly, without replacing the wedge ring, if the ring is made of a relatively hard material which

will elastically recover on release of pressure. An example of this application is in vessel closures which must be opened and closed between pressure cycles.

As shown in Fig. 14B, a rubber O-ring is usually used to provide the initial seal in this design. However, other types of initial seals and combinations of O-rings and back-up rings can be used depending on the specific applications.

The selection of the material for the final seal ring in the Bridgman seal (Fig. 14A) or wedge ring (Fig. 14B) is based on a compromise. It must have the ability to plastically deform sufficiently to conform to the sealed surfaces but must have sufficient hardness to resist extrusion into the clearance between the closure and cylinder. At relatively low pressures such materials as nylon, mild steel, copper and various brasses and bronzes are commonly used. At higher pressures beryllium copper and hardened aluminum alloys are suitable for both dynamic and static conditions. For static seals, nickel alloys such as monel are very useful due to their very high strain hardening capabilities thus retarding extrusion. In selecting a material for this application, consideration must also be given to the possibility of "galling" or cold welding of the ring and cylinder materials.

For specific applications, one might consider combining various features of the two designs. For example, as proposed by Newhall[52], wedge rings can be used as anti-extrusion rings in the Bridgman seal in very high pressure applications.

Another method of obtaining the effect of a high pressure seal, without any actual seal in the usual sense, is the controlled clearance principle. If some method can be devised to control the clearance between a finely lapped piston and cylinder at some very small value in the order of a few microinches, the fluid leakage, even at pressures up to 15 kb, can be maintained at tolerable levels. This principle permits the design of very low friction piston and cylinder devices. This controlled clearance can be accomplished by applying a separately controlled external hydrostatic or mechanical pressure on the outside of the cylinder. By observing the leakage rate and the frictional force (usually by rotating the piston) simultaneously, this external pressure, and thus the clearance, can be maintained at a minimum possible value.

For high temperature applications, one must insure that the initial sealing material can withstand the temperatures involved. At very high temperatures, it may be necessary to use an all metal seal. This may be accomplished by using a mechanical force to obtain the initial seal although this may result in the problems mentioned previously. A series of progressively softer metallic rings may sometimes be used in place of the elastomer seal. There are also some commercial, all metal initial seals available such as metal O-rings or metal lip seals.

The most commonly used electrical insulating seal was again discussed by Bridgman[50] after a suggestion by Amagat and is shown in Fig. 15. The electrode is a hardened tool steel with electrical leads

resistance welded or soldered to both ends. The insulating cone may be of a number of materials depending on the pressure and operating conditions. For pressures up to about 10 kb, a plastic such as nylon can be used. The greatest pressures can be obtained by making this plastic cone as thin as possible without shorting the electrode to ground. If the insulating cone is constructed of pyrophyllite or certain ceramics, this seal can be used up to pressures of 30 kb or greater.

In practice, one usually requires a number of electrical leads passing into the pressure chamber. This can be accomplished by having a multiplicity of electrodes of the type shown in Fig. 15 that are completely independent or have a common port on the atmospheric pressure side as proposed by Pugh[6]. Another approach is that proposed by Blosser and Young[53], as shown schematically in Fig. 16 wherein multiple leads are introduced by being imbedded in a molded epoxy shell which acts as the insulator. Such a configuration is reported to work well to pressures of 10 kb.

There are, of course, many other insulated seal designs in addition to those cited above. However, most are limited to pressures below approximately 10 kb. The Bridgman type appears to be the most reliable over the entire pressure range of concern.

Only a few of the many possible and workable seal designs for high pressure applications have been discussed. There is a great number of ingenious designs developed by various designers and researchers to suit their particular needs. However, the basic designs presented

herein, with modifications and in combination with standard practices from the lower pressure hydraulics technology, can be used to solve virtually all high pressure seal problems.

V. SUPPORT OF END CLOSURES

At this stage, attention should be drawn to an aspect of high pressure vessel design to which inadequate consideration is often given. This is the problem of supporting the end closures of the vessel. In many very high pressure vessels, the end closures are supported by an external press or frame, as shown in Fig. 10, which carries the force produced by the action of the pressure on the end closures. In this case, the problem is to design a press which will support this force without excessive deflection that could cause seal failure.

The problem of primary concern is the case of the closed end vessel in which the end load on the closures is supported by the walls of the vessel itself. Design errors in the region of the end closure attachment point have been the prime cause of the majority of unexpected, catastrophic failures of very high pressure vessels.

The problems with end closure support usually arise from the combined effects of the uniform longitudinal stress resulting from the end force, the longitudinal bending stresses resulting from the pressure discontinuity at the seal, and the stress concentrations arising from geometrical factors involved with the seal and the closure attachment. The radial and tangential stresses produced by the pressure itself may also contribute. As before, a discussion of specific designs is not considered appropriate or practical. However, a general discussion of some of the significant factors involved will follow.

Seal Location - In order to avoid as much interaction of the above stresses as possible, the closure attachment point should be located some distance removed from the point of seal.

Stress Concentrations - All stress concentrations such as cross holes, grooves, keyways and threads should be removed from the immediate area of the point of seal. If geometric discontinuities cannot be avoided in this area, generous radii should be provided at all re-entrant corners.

Threads - The most common method of closure attachment is by means of threads on the cylinder. Although this chapter is not intended to be a text on thread design, the following points, unique to pressure vessel design, should be noted.

Avoid fine threads. They are subject to failure resulting from reduction of bearing area due to radial dilation of the cylinder. Fine threads cannot be made with sufficiently large root radii to avoid severe stress concentrations. Also, they have a strong tendency to gall in pressure vessel application although this problem can be somewhat reduced by using interrupted threads.

In multi-layered cylinders, the cylinder threads should generally not be machined into the inner cylinder. This will usually result in the entire longitudinal force on the end closure being supported by the inner cylinder in the region of the seal, thus producing very high longitudinal stresses in this region. To eliminate this problem, the closure should be supported by one of the outer

elements, preferably the outer-most. It is also preferable to place the threads on the outside surface of the supporting element since this surface is less highly stressed.

In designing a thread in a vessel, consideration must be given to the effect of the radial component of the bearing stress between the bearing faces of the thread. In a relatively thin-walled cylinder, this can result in high radial dilations and high tangential stresses. For this reason a "buttress" thread is often used for this application. The standard 7 degree flank angle buttress thread, however, produces high bending stresses at the thread root. Recent studies[54] have shown a modified buttress thread with a 20 degree flank angle and a generous root radius to be an optimum thread design for many pressure vessel applications.

The foregoing discussion is not intended to be a complete review of the state-of-the-art in high pressure equipment or a survey of commercially available systems and components. It is intended only to present the basic design principles applicable to the design of any high pressure system. For a detailed discussion of specific designs and commercial suppliers of high pressure equipment, the reader is referred to References 55, 28, 45 and 48.

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TABLE I PRESSURE LIMITS AND DESIGN EQUATIONS FOR VARIOUS CYLINDER CONFIGURATIONS

CYLINDER CONFIGURATION	ASSUMPTIONS	THEORETICAL ELASTIC PRESSURE LIMIT	PERTINENT DESIGN EQUATIONS
MONOBLOC		$0.5\ \sigma_y$	$\frac{P_y}{\sigma_y} = \frac{K^2 - 1}{2K^2}$
MULTI-LAYER (2 ELEMENT)	ELEMENTS OF EQUAL STRENGTH	σ_y	$\frac{P_y}{\sigma_y} = 1 - \frac{1}{K_t}$
MULTI-LAYER (m ELEMENTS)	ELEMENTS OF EQUAL STRENGTH AND EQUAL DIAMETER RATIO	σ_y	$\frac{P_y}{\sigma_y} = \frac{m}{2}(1 - \frac{1}{K_t^{2/m}})$ OR $\frac{P_y}{\sigma_y} = \frac{K_t^2 - 1}{K_t^2}$
AUTOFRETTAGED	COMPLETE OVERSTRAIN ($K \leq 2.22$) 100% AUTOFRETTAGE ($K \geq 2.22$)	σ_y	$\frac{P_y}{\sigma_y} = \ln K$ ($K \leq 2.22$) $\frac{P_y}{\sigma_y} = \frac{K^2 - 1}{K^2}$ ($K \geq 2.22$)

TABLE I (Continued)

CYLINDER CONFIGURATION	ASSUMPTIONS	THEORETICAL ELASTIC PRESSURE LIMIT	PERTINENT DESIGN EQUATIONS
VARIABLE DIRECT EXTERNAL PRESSURE OR TAPERED OUTER CYLINDER	σ_y(LINER) = σ_y(OUTER CYLINDER) K_1(LINER) = K_2(OUTER CYLINDER) $\geq$ 2.22 AUTOFRETTAGED LINER AND OUTER CYLINDER	$2\ \sigma_y$	$\frac{P_y}{\sigma_y} = \frac{2(K_t - 1)}{K_t}$
SEGMENTED			
SOLID OUTER CYLINDER	σ_y(SEGMENTS) = $2\sigma_y$(OUTER CYLINDER) K_2(OUTER CYLINDER) = 2.22 AUTOFRETTAGED OUTER CYLINDER	$3\ \sigma_y$	$\frac{P_y}{\sigma_y} = 0.359\ K_t$ OR $\frac{P_y}{\sigma_y} = 2\ (1 + \alpha)$
THIN-LINER - VARIABLE EXTERNAL PRESSURE	σ_y(SEGMENTS) = $2\sigma_y$(OUTER CYLINDER) K_2(OUTER CYLINDER) = 2.22 AUTOFRETTAGED OUTER CYLINDER	$3\ \sigma_y$	$\frac{P_y}{\sigma_y} = 0.359\ K_t$ OR $\frac{P_y}{\sigma_y} = 2\ (1 + \alpha)$
THICK-LINER - VARIABLE EXTERNAL PRESSURE	σ_y(SEGMENTS) = $2\ \sigma_y$(LINER) = $2\ \sigma_y$(OUTER CYLINDER) K_1(LINER) $\geq$ 2.22 K_3(OUTER CYLINDER) = 2.22 AUTOFRETTAGED LINER AND OUTER CYLINDER	$4\ \sigma_y$	$\frac{P_y}{\sigma_y} = 0.359 \frac{K_t}{K_1} + \frac{K_1^2 - 1}{K_1^2}$ OR $\frac{P_y}{\sigma_y} = 2(1 + \alpha) + \frac{K_1^2 - 1}{K_1^2}$

TABLE II TYPICAL COMPOSITION OF VARIOUS ALLOYS

ALLOY	C	Mn	Si	Ni	Cr	V	Mo	W	Co	Al	Ti	Cb	Fe
4140	0.40	.85	0.30	—	0.95	—	0.20	--	--	--	--	—	BAL
4340	0.40	.70	0.30	1.83	0.80	—	0.25	--	--	--	--	--	"
250 MARAGING	0.03	.12	0.12	18.0	—	—	4.8	--	8.0	0.10	0.40	--	"
300 MARAGING	0.03	.12	0.12	18.0	—	--	5.0	--	9.0	0.10	0.55	--	"
H.P.-Ni-Co	0.28	.25	0.10	8.00	0.50	0.10	0.50	--	3.75	--	--	--	"
S5	0.060	.070	1.85	--	--	0.20	0.45	--	--	--	--	--	"
S7	0.050	.070	0.25	—	3.25	--	1.40	--	--	--	--	--	"
HTB-2	0.30	.25	0.15	0.10	4.00	1.00	4.25	0.10	0.25	--	--	--	"
VERSATOOL	1.00	.30	2.00	--	4.25	1.15	2.50	0.30	--	--	--	--	"
RENÉ 41	0.09	--	--	BAL	19.0	--	10.00	--	11.00	1.5	3.1	--	--
INCO 718	0.04	.20	0.30	"	18.6	--	3.1	--	--	0.40	0.9	5.00	13.5
WASPALOY	0.08	--	--	"	19.5	—	4.3	--	13.5	1.30	3.0	--	--

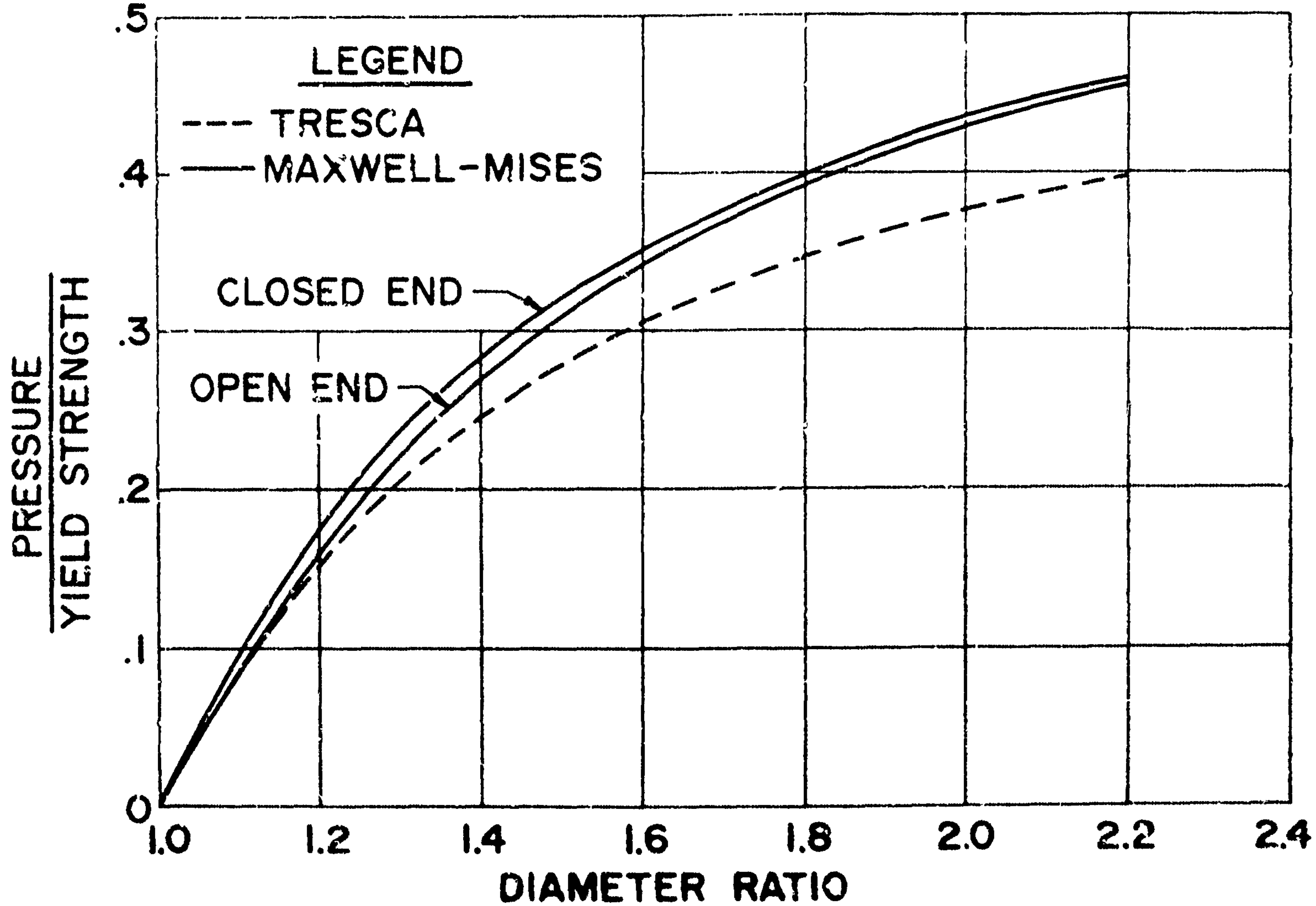


Figure 1. Initial Yield Pressure vs. Diameter Ratio Based on Various Yield Criteria.

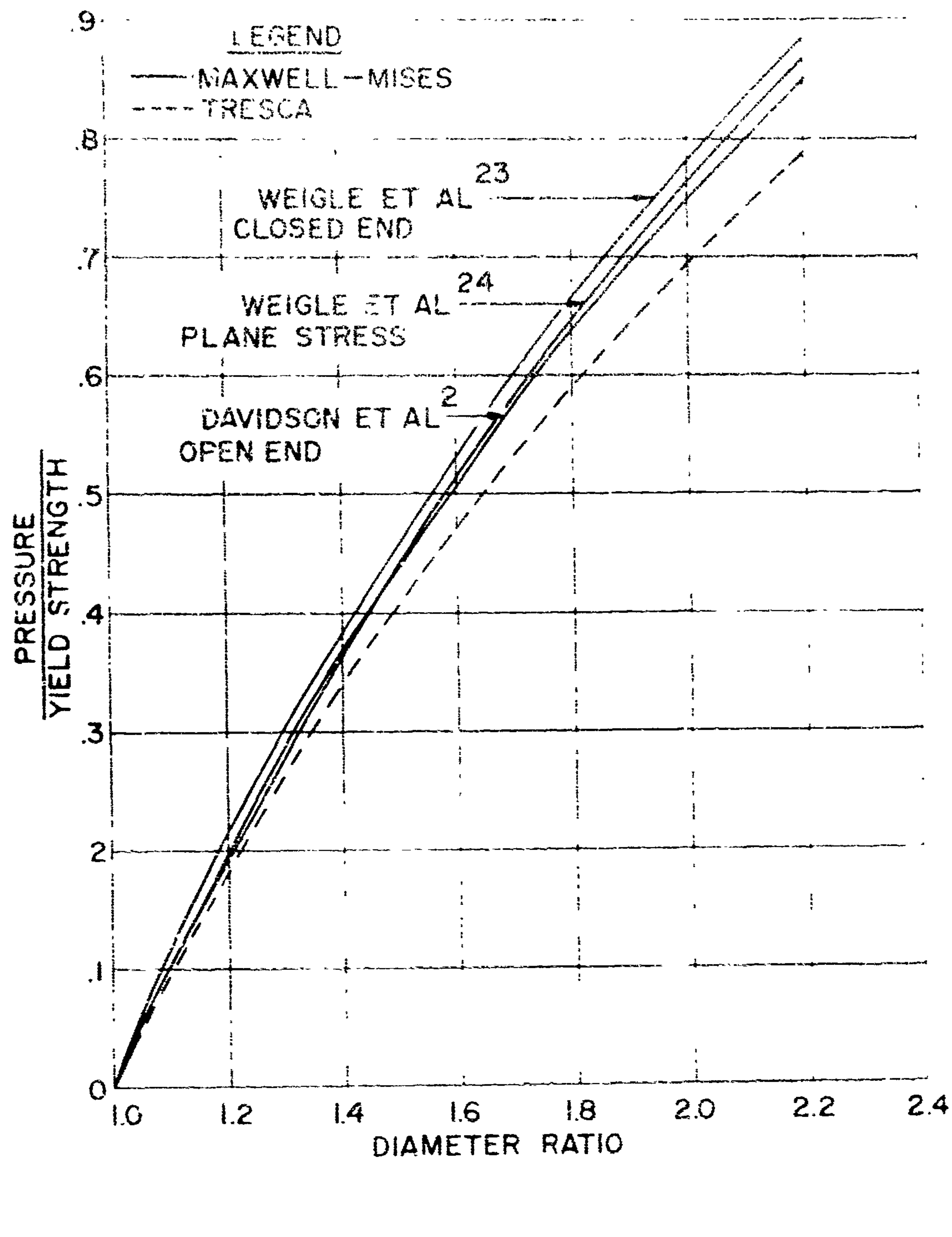


Figure 2. Complete Overstrain Pressure vs. Diameter Ratio Based on Various Theories.

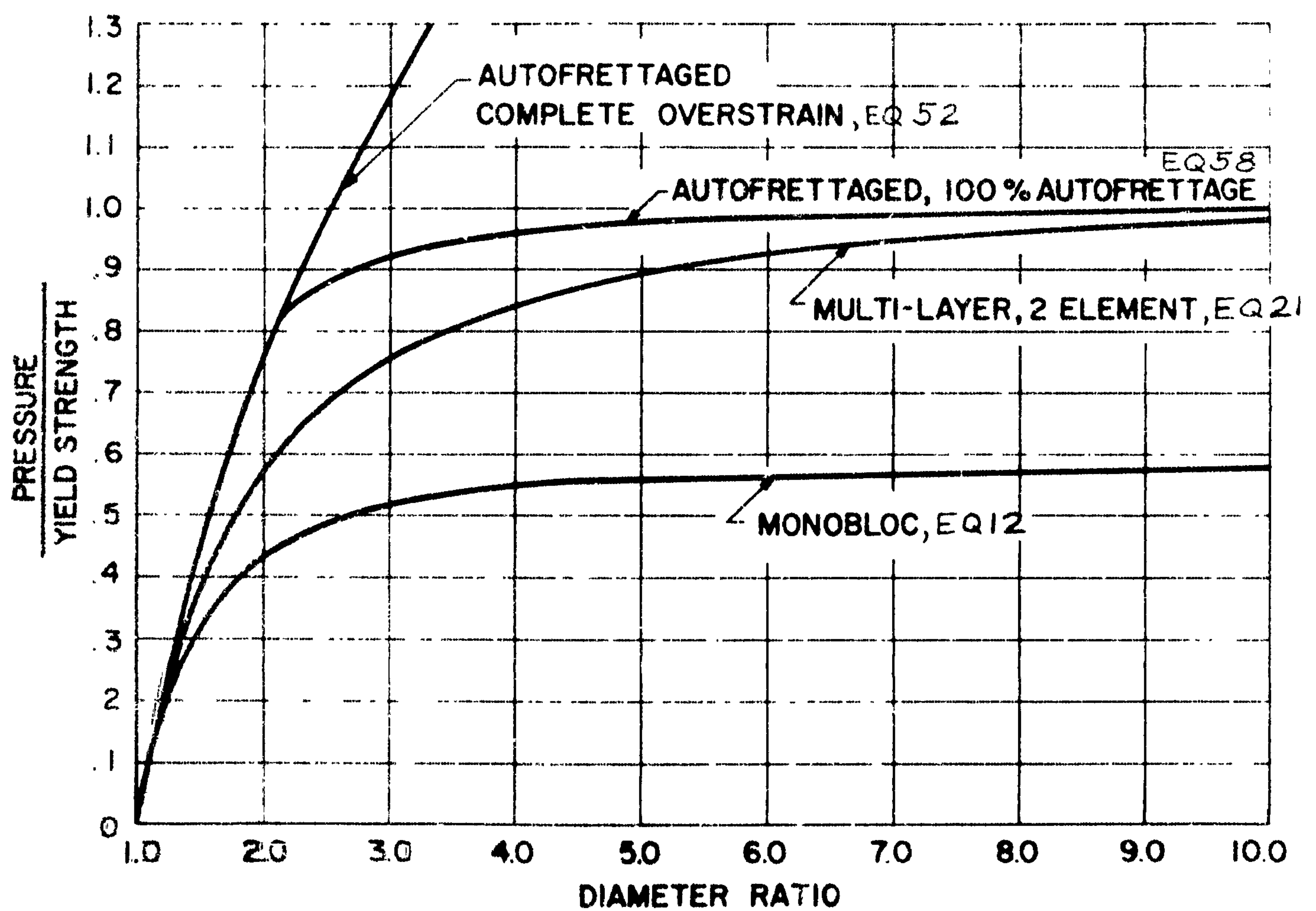


Figure 3. Comparison of Monobloc, Multi-layer and Autofrettaged, Open-end Cylinders Based on Maxwell-Mises Yield Criterion.

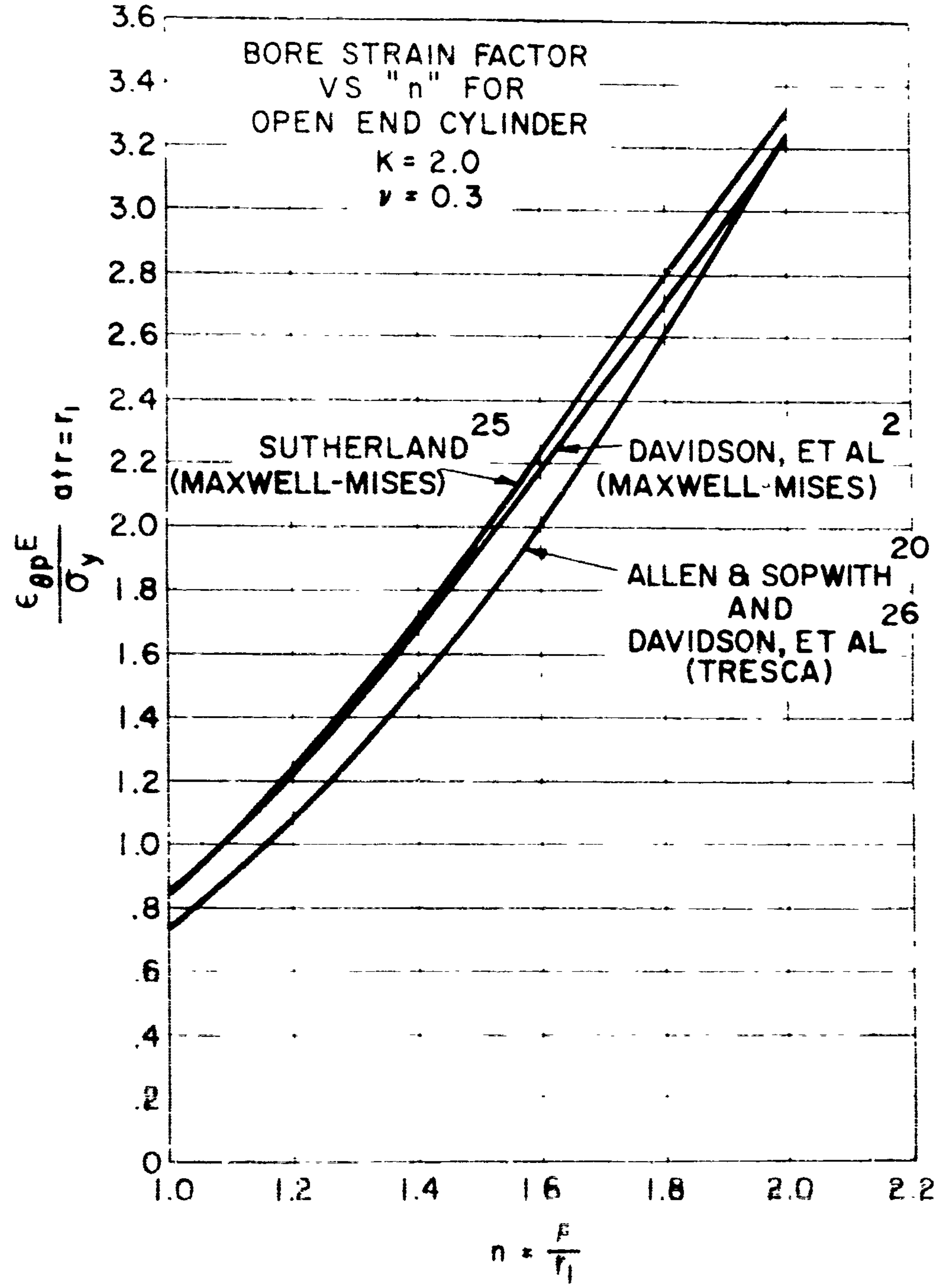


Figure 4. Bore Strain vs. n For Open-end Cylinders by Various Theories.

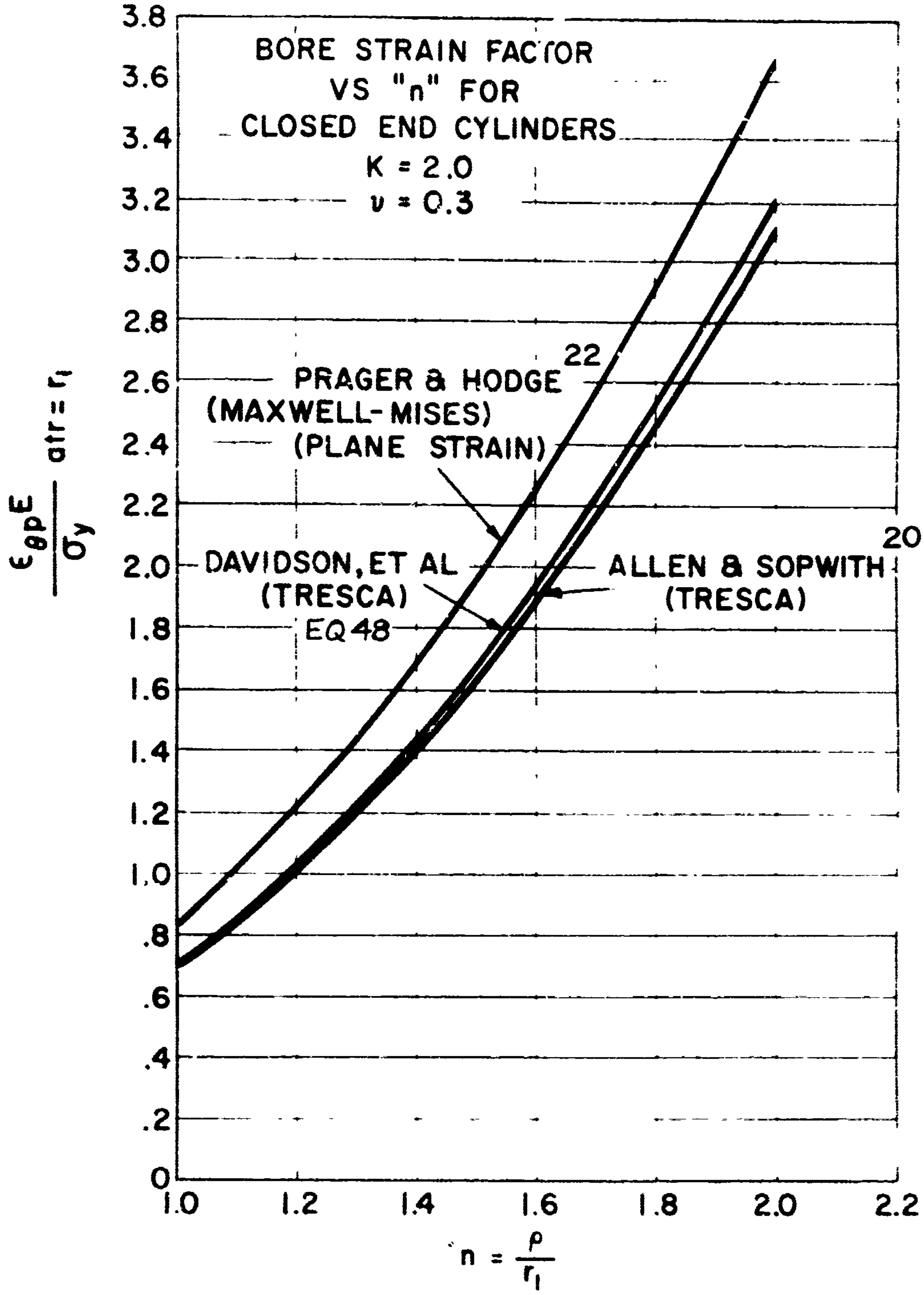


Figure 5. Bore Strain vs. n For Closed-end Cylinders by Various Theories.

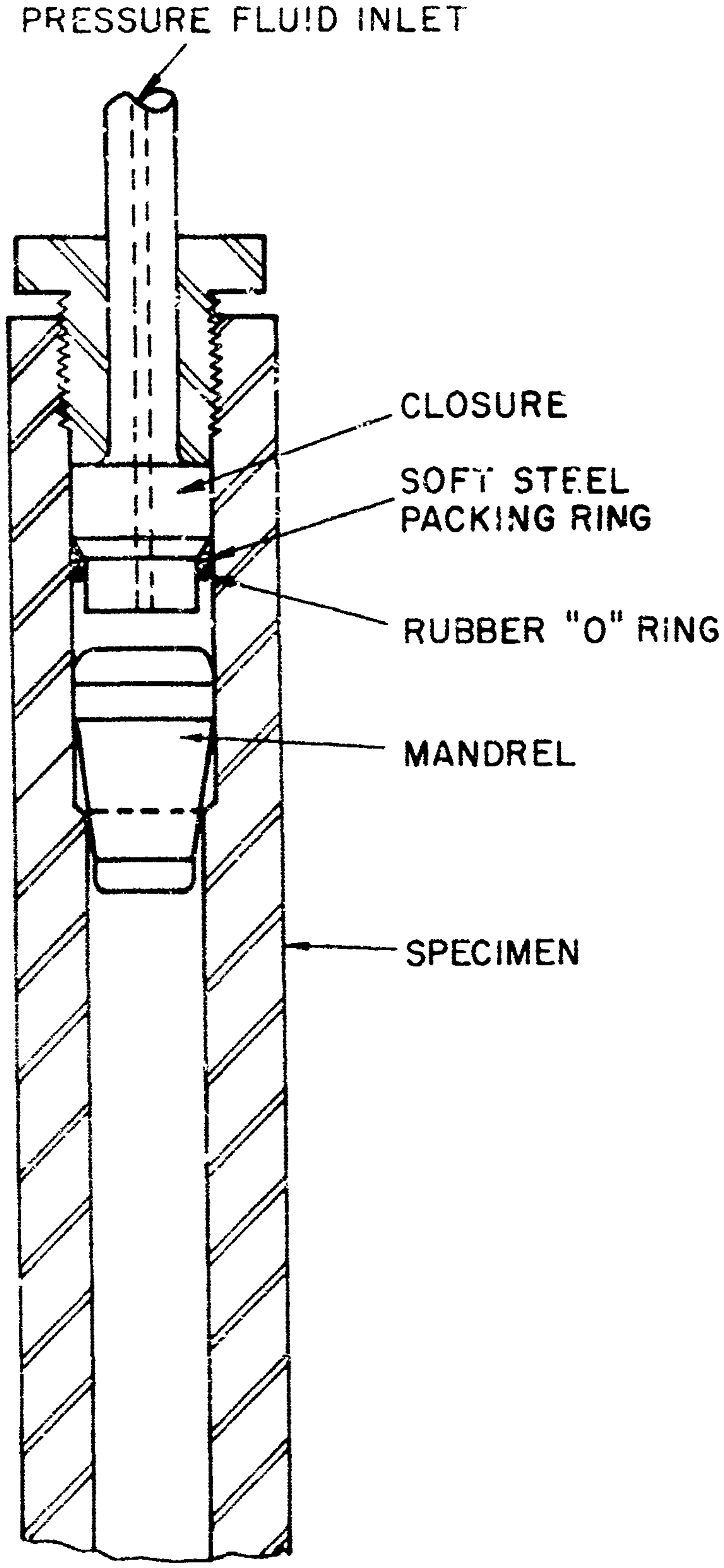


Figure 6. Mechanical Autofrettage Process.

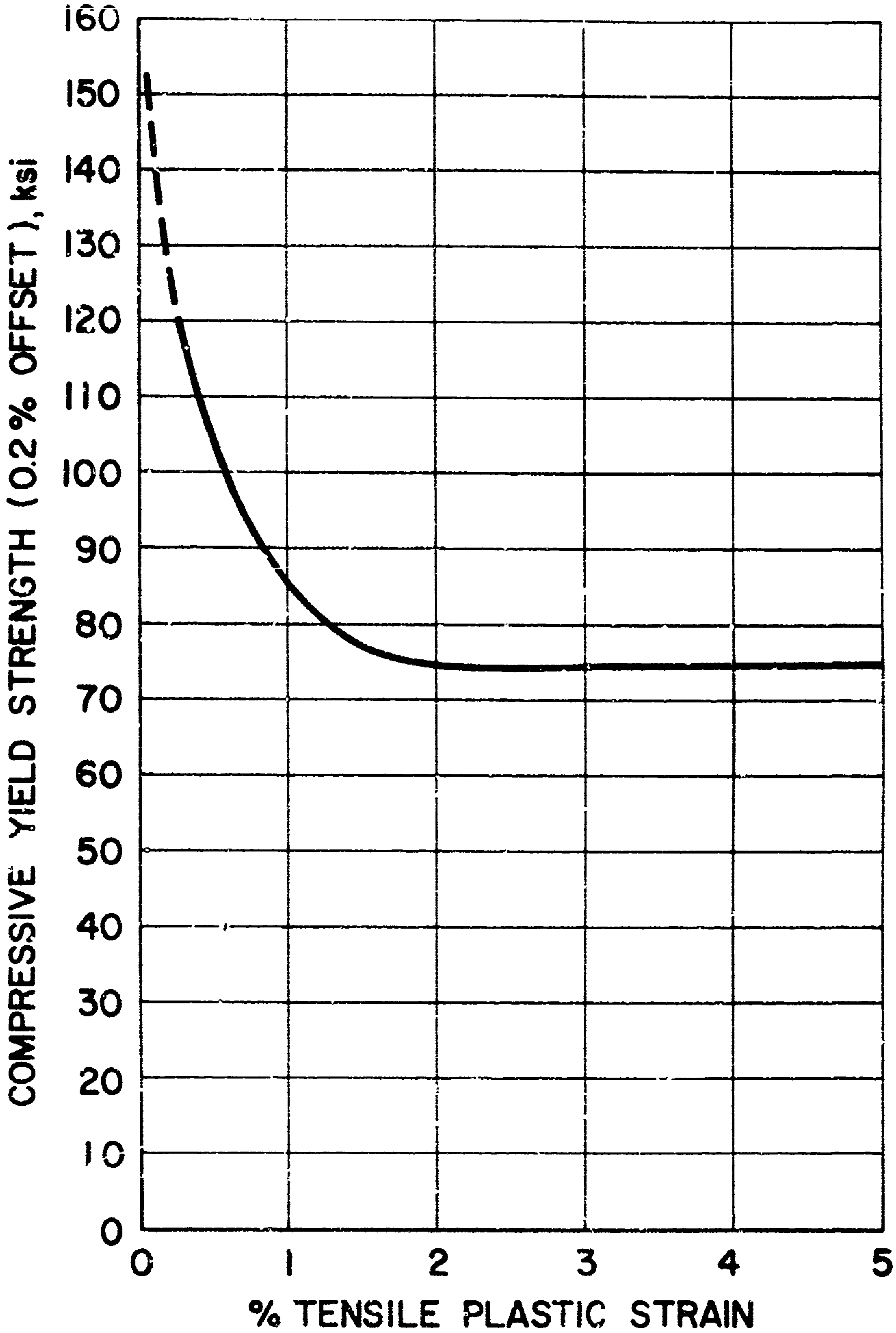


Figure 7. Compressive Yield Strength vs. Tensile Plastic Strain Showing Bauschinger Effect.

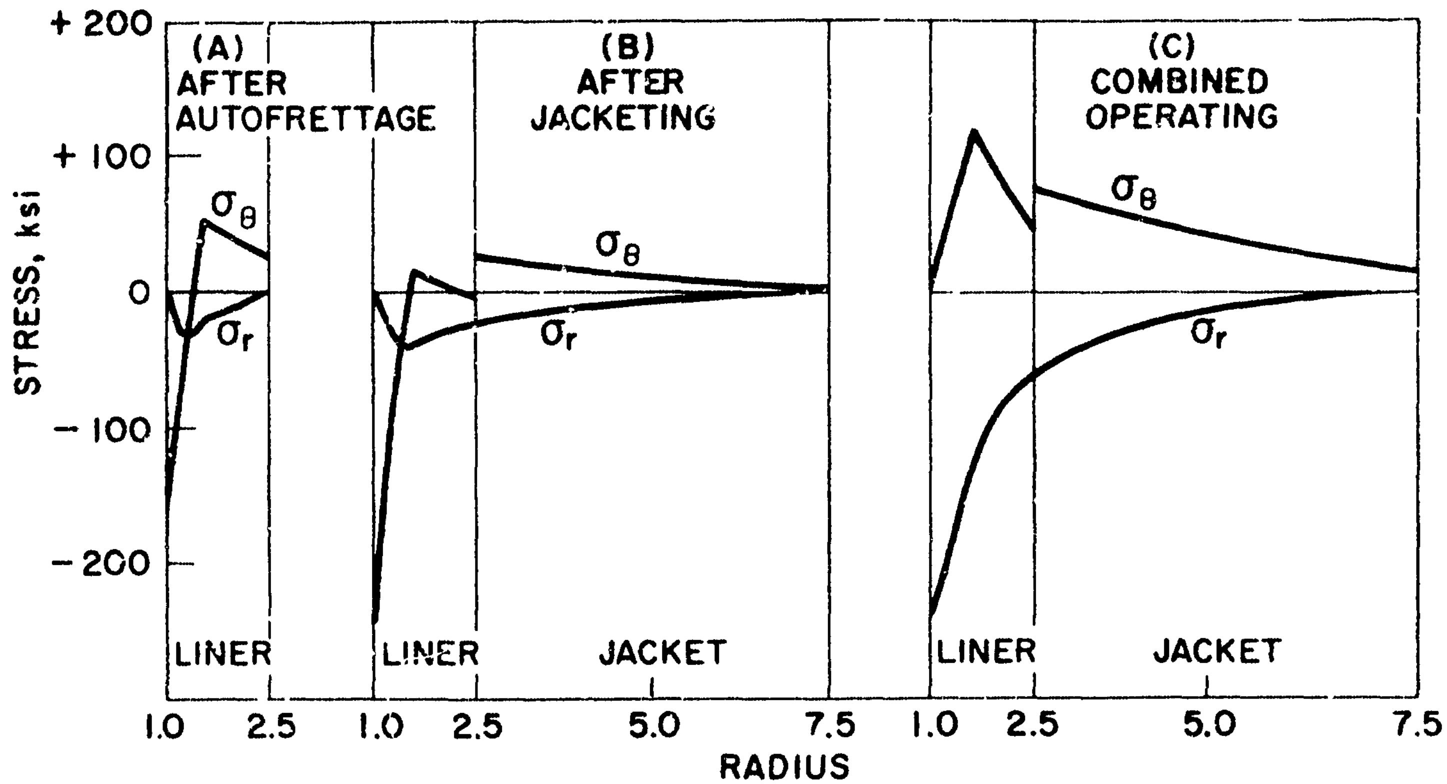


Figure 8. Summation of Stresses in Combination Autofrettaged and Jacketed Cylinder.

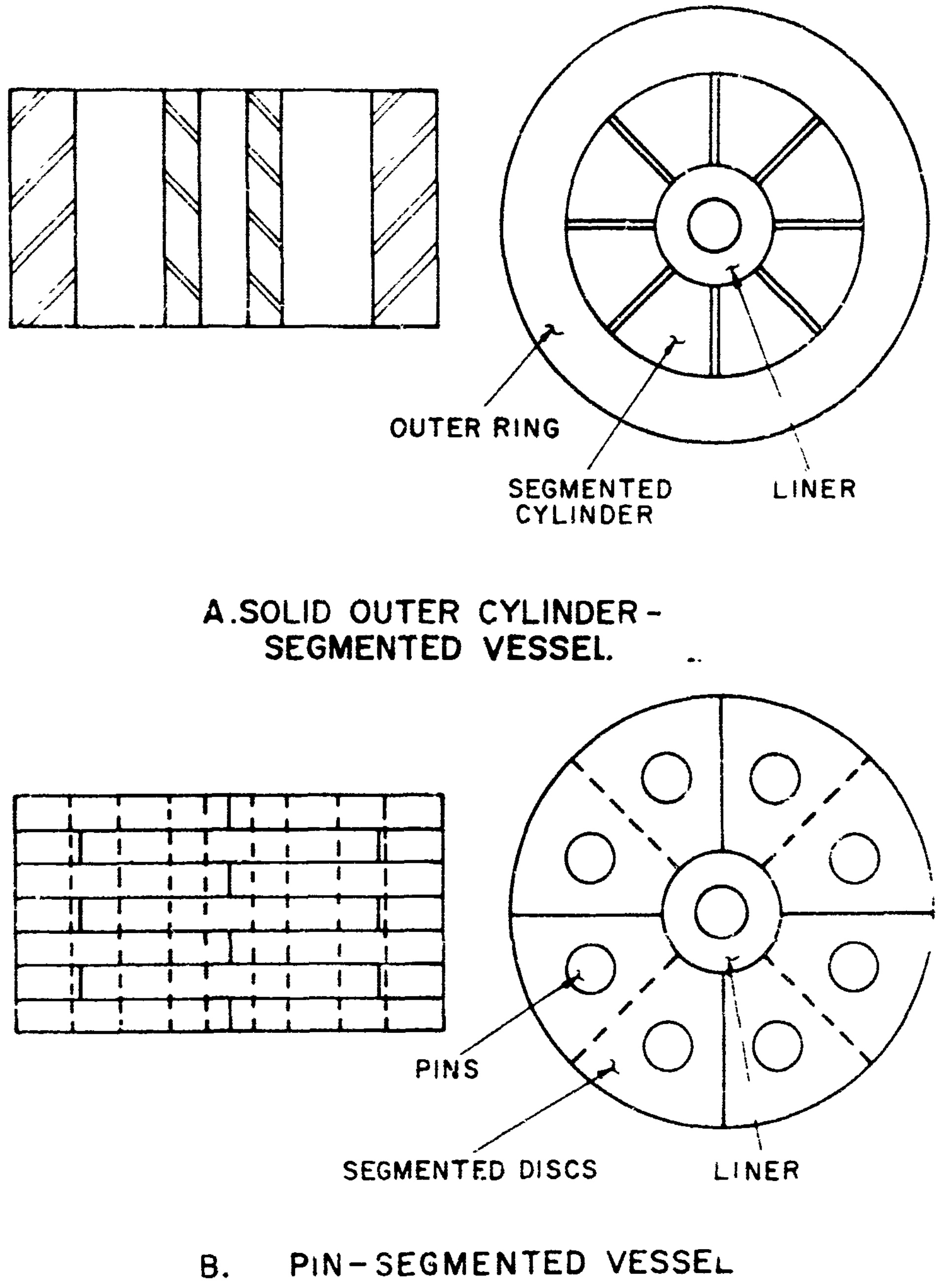


Figure 9. Solid Outer Cylinder-segmented and Pin-segment Vessel Designs.

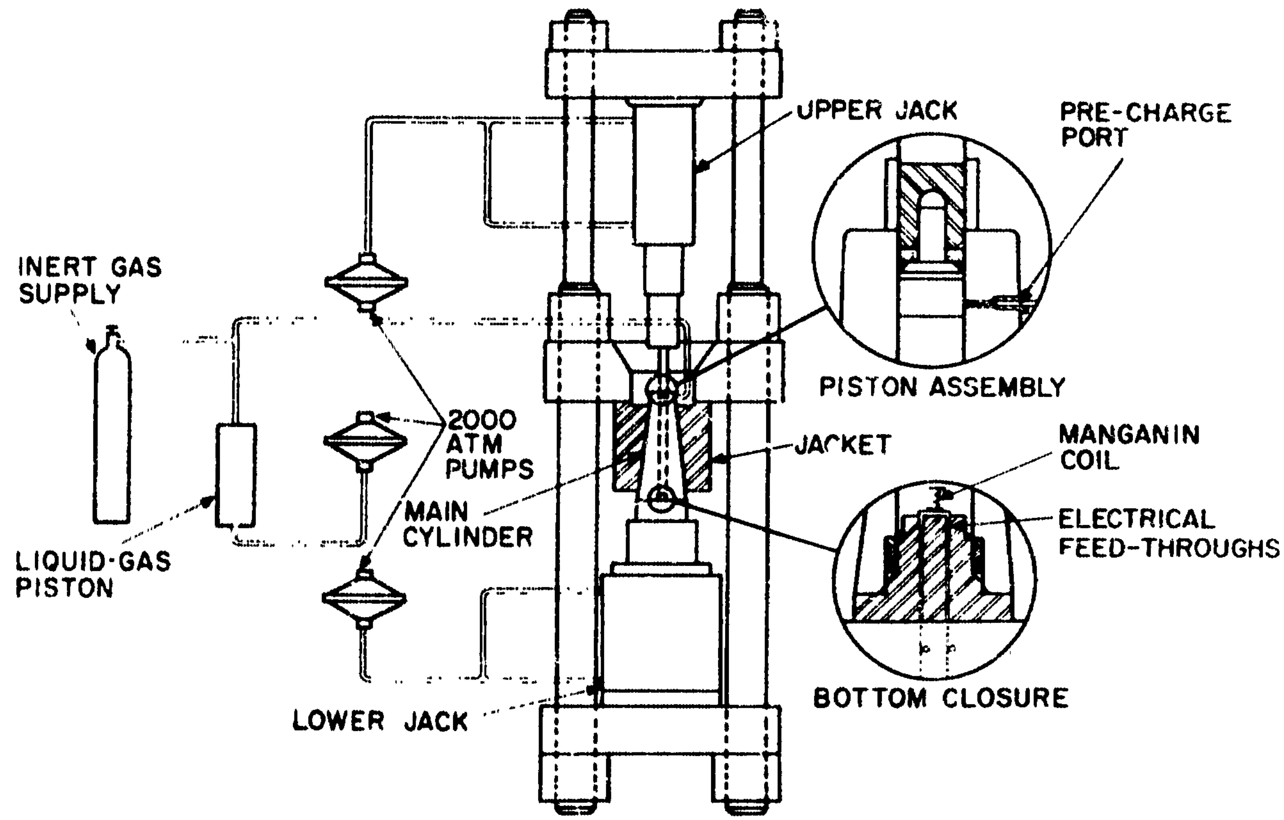


Figure 10. Tapered External Cylinder Vessel System.

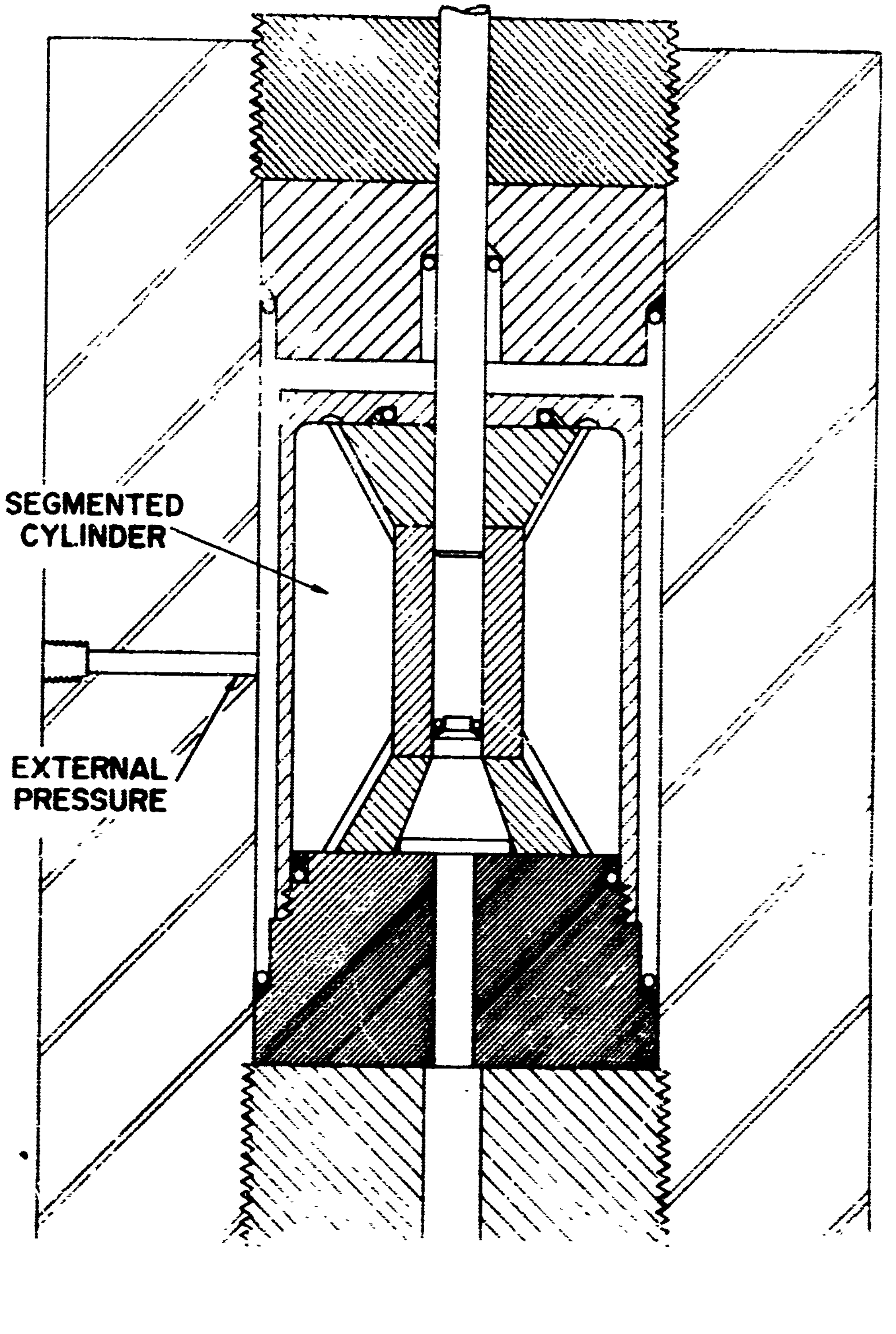


Figure 11. Segmented Cylinder Vessel Using Variable External Pressure.

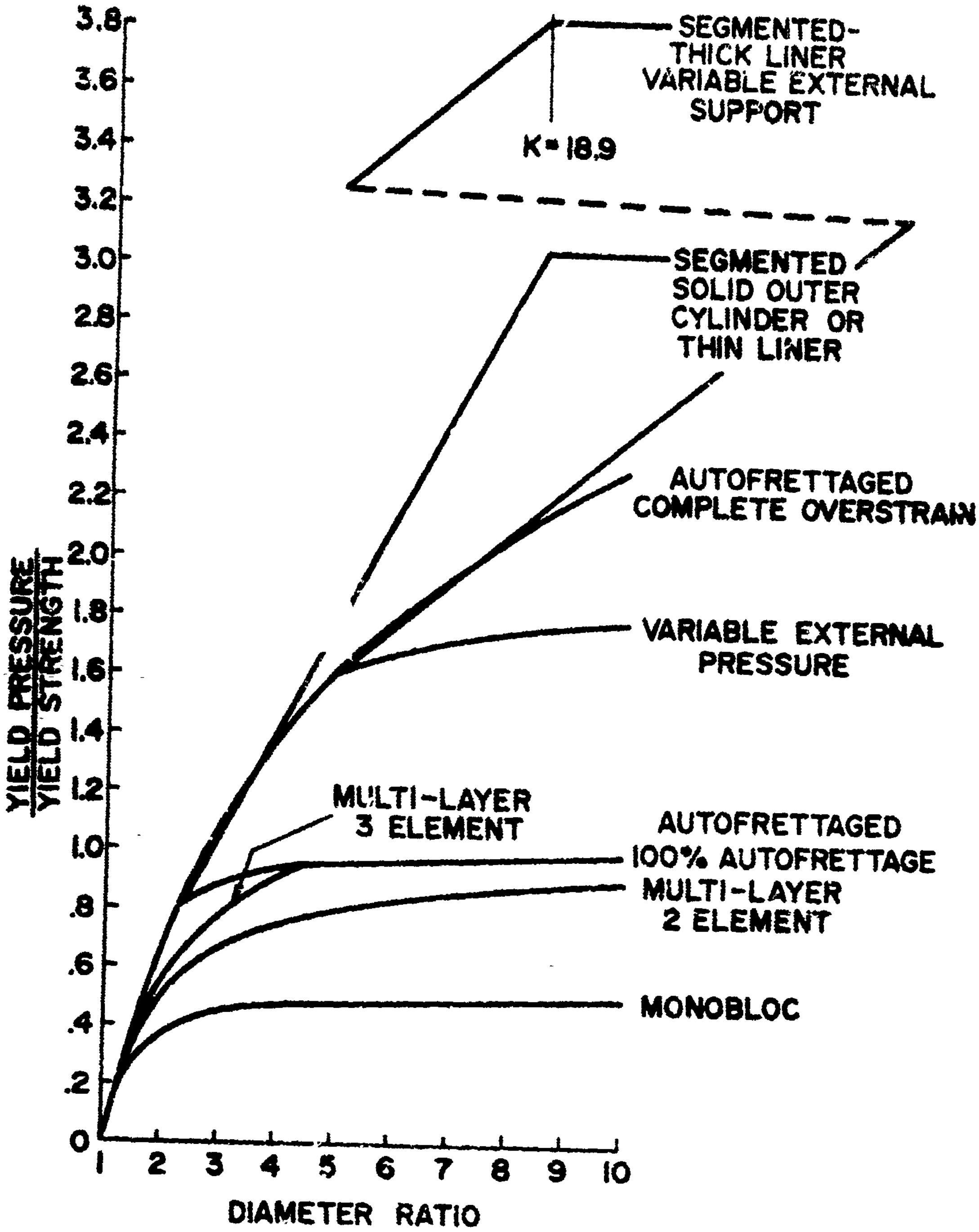
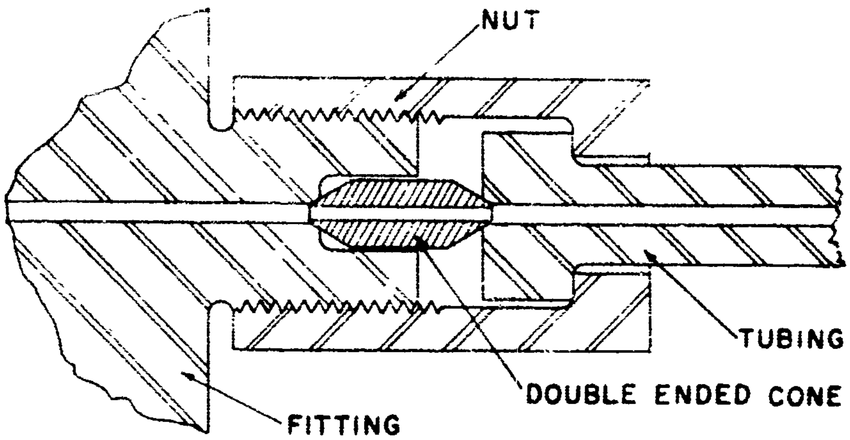
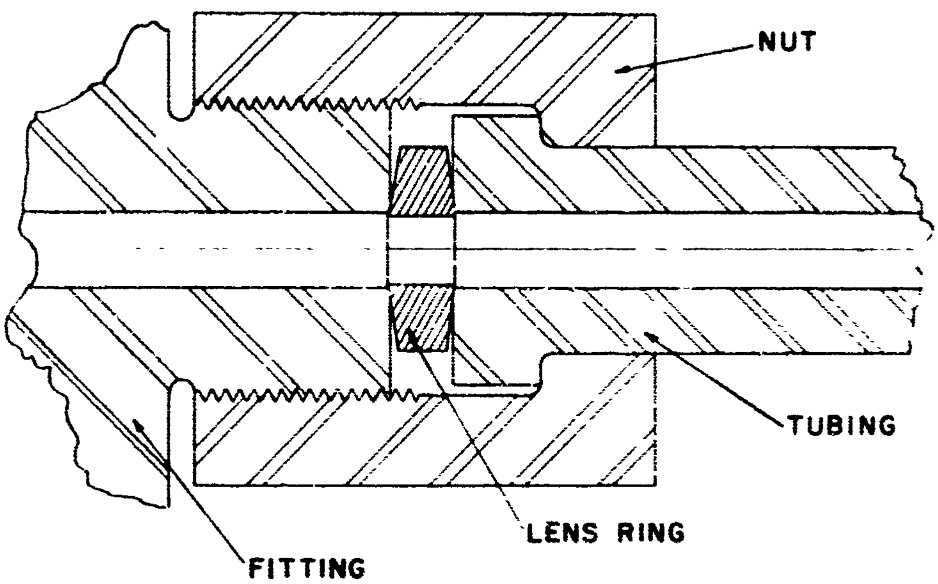


Figure 12. Comparison of Various Pressure Vessel Designs Based on the Tresca Yield Criterion.



A. CONE SEAL



B. LENS RING SEAL

Figure 13. Cone and Lens Ring Seals.

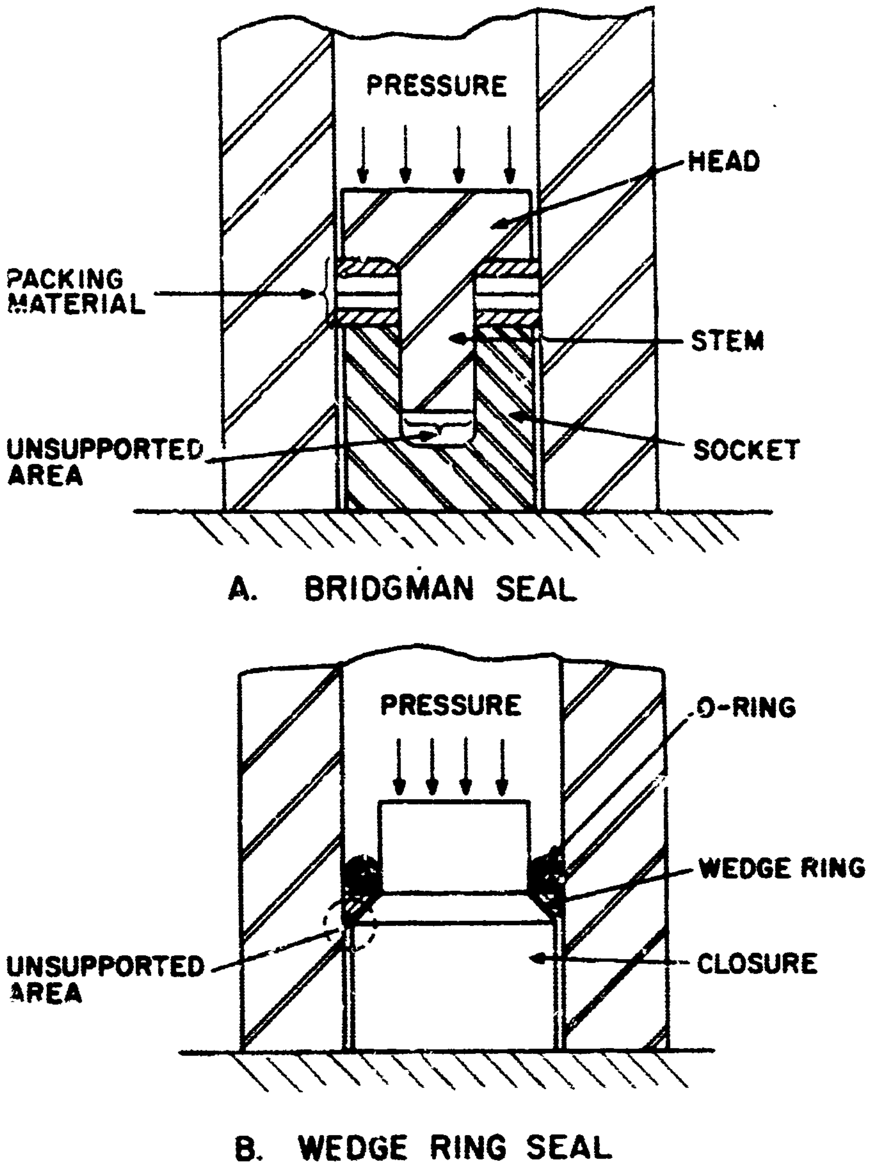


Figure 14. Bridgman Unsupported Area and Wedge Ring Seals.

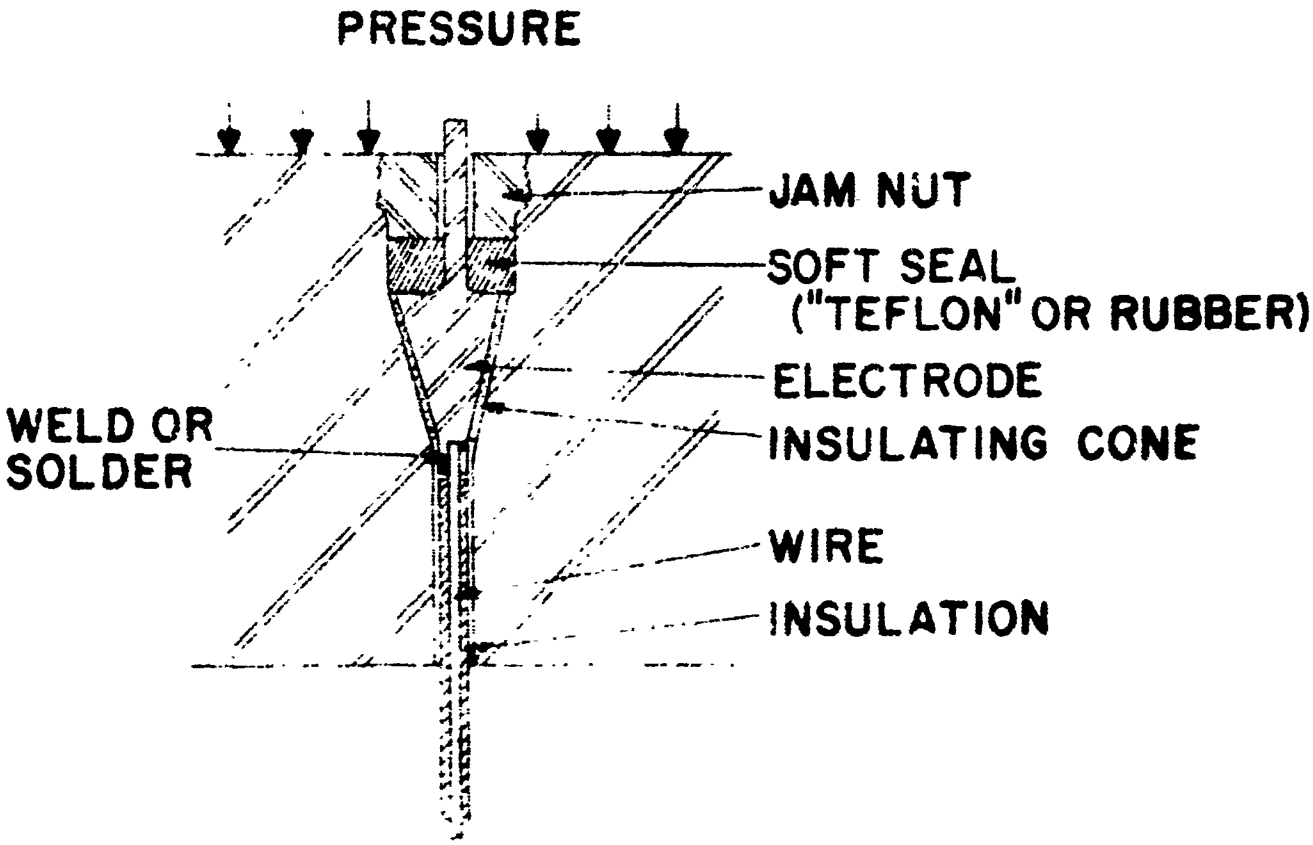
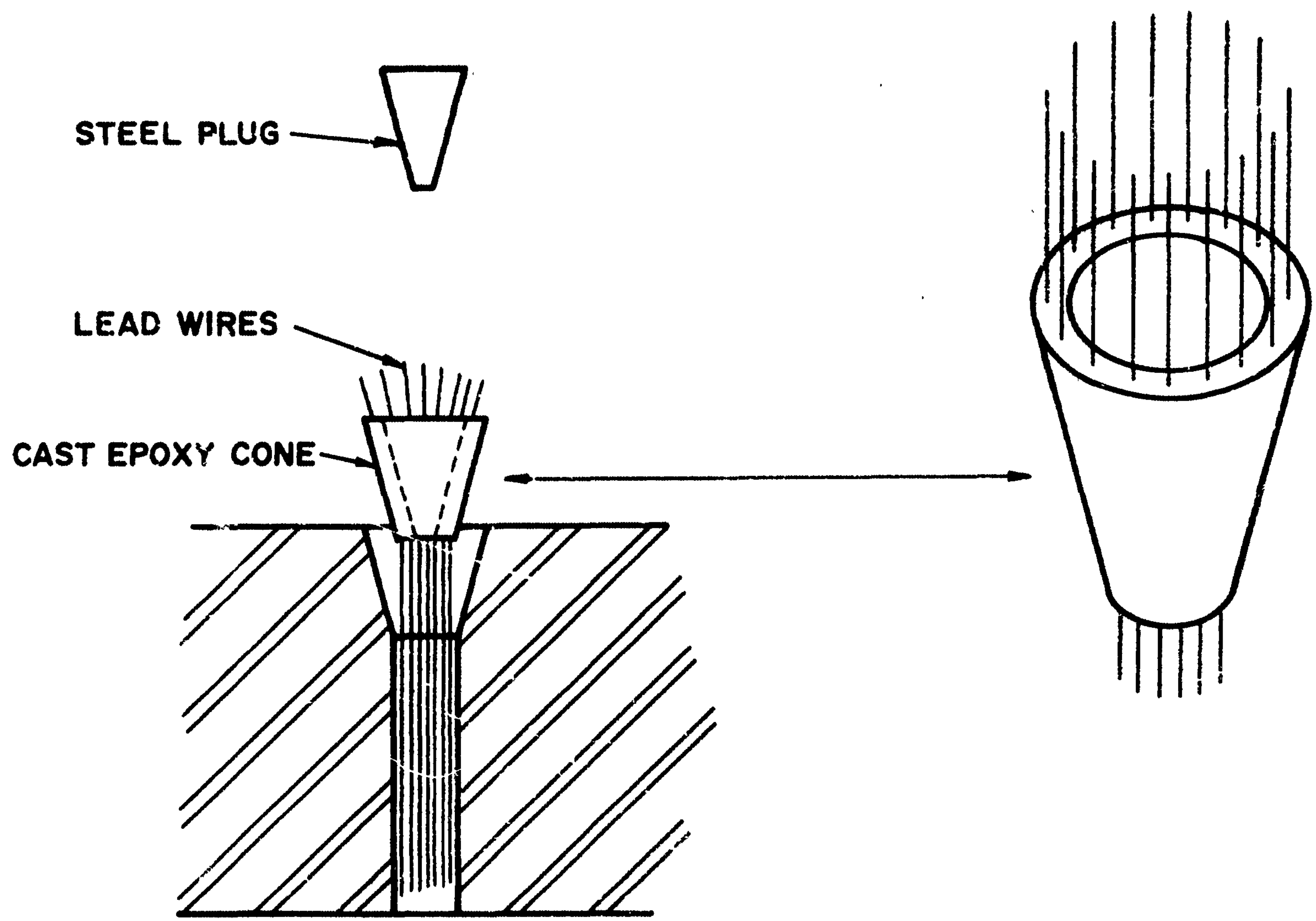


Figure 15. Electrically Insulated Seal.



· Figure 16. Multiple Lead Insulated Seal.

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13. ABSTRACT

This report is a review of the theory and practice of pressure vessel design for vessels operating in the range of internal pressures from 1 to 55 kilobars (approximately 15,000 to 800,000 psi) and utilizing fluid pressure media. The fundamentals of thick walled cylinder theory are reviewed, including elastic and elastic-plastic theory, multi-layer cylinders and autofrettage. The various methods of using segmented cylinders in pressure vessel design are reviewed in detail.

The factors to be considered in the selection of suitable materials for pressure vessel fabrication are discussed. These factors include strength, toughness and environmental factors. A brief review of the materials currently available is also included.

The report also includes a discussion of pressure seals and closures suitable for use in this pressure range and of methods of supporting the end closures of the vessel.

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Pressure seals						